New Developments in Distance Relay Polarization and Fault Type Selection

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INTRODUCTION

Single-pole tripping, targeting, and signal processing economy require fault type selection. Past methods of distance relay element polarization have compromised between sensitivity and fault-type selectivity. For example, the stronger the amount of cross-polarization, the more likely it is that unwanted distance relay elements respond to a given fault type. Now, a new technique for fault-type selection is so effective that positive-sequence voltage polarization with memory can be used. The key to the new technique is analysis of the products or "torques" produced by mho relay elements AG, BG, CG, AB, BC, and CA. In general, fault type corresponds to the element producing the strongest torque.

Torque-comparison and the positive-sequence memory filter were straightforward to implement in a microprocessor relay design. Indeed, microprocessor relay technology makes it possible to consider and use the new techniques described in this paper.

This paper includes:

- Mho characteristics for positive-sequence voltage polarization, with and without memory
- A simple scheme for determining positive-sequence memory polarization quantities
- Principle of mho-element torque-comparison for fault type determination
- Simulation studies demonstrating the torque-comparison principle

DISTANCE RELAY POLARIZATION QUANTITY REQUIREMENTS

Figure 1 shows the familiar development of the self-polarized mho circle characteristic, where $Z_i$ is the diameter of the relay characteristic, $I$ is the current in the relay, and $V$ is the voltage presented to the relay. The relay forms a difference quantity:

$$dV = Z_i^* I - V,$$

and tests the phase relationship between $dV$ and $V$. If the angle is greater than 90 degrees, the impedance $V/I$ is inside the circle. Otherwise, it is on or outside the circle.

If the voltage $V$ is zero, then there is no reference voltage against which to compare the difference voltage $dV$ and the relay element is not secure: the angle of a zero-length phasor is undefined.

Two common methods overcome this problem:

- Memory polarization
- Combinations of other phase voltages
For instance, the mho element for AG faults could be polarized by jVBC instead of VA. Then the AG element would still be positively polarized for any AG fault, even a solid one at the bus.

Polarizing the mho element with memory or sound-phase voltages expands the mho circle characteristic, as Figure 2 shows.

The choice of polarizing voltage affects the general requirements of the distance relay elements listed below:

1. Provide reliable operation for all in-zone faults including close-in faults.
2. Be secure for all external faults.
3. Be compatible with fault-type selection logic and schemes.
4. Provide stable operation during single-pole-open load conditions.
5. Tolerate fault resistance.
6. Be easy to implement.

**POSITIVE-SEQUENCE VOLTAGE POLARIZATION WITH MEMORY GIVES MAXIMUM EXPANSION**

**Relay Characteristics and Equations**

When we use positive-sequence memory voltage polarization, the mho characteristics expand all the way back to the source. The relay impedance setting $Z_0$ defines the maximum reach point. Figures 3, 4 and 5 show the impedance plane characteristics for several fault types. The forward-characteristic mho circles extend from the source impedance $Z_S$, forward to the relay impedance setting $Z_0$.

Table 1 shows the voltage and current combinations used to operate and polarize the six mho elements.
Table 1: Voltages and Currents for Mho Elements AG, BG, CG, AB, BC, CA

<table>
<thead>
<tr>
<th>Element</th>
<th>Voltage</th>
<th>Current</th>
<th>Polarization</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG</td>
<td>VA</td>
<td>IA + K * IR</td>
<td>VA1m</td>
<td>Tag</td>
</tr>
<tr>
<td>BG</td>
<td>VB</td>
<td>IB + K * IR</td>
<td>VB1m</td>
<td>Tbg</td>
</tr>
<tr>
<td>CG</td>
<td>VC</td>
<td>IC + K * IR</td>
<td>VC1m</td>
<td>Tcg</td>
</tr>
<tr>
<td>AB</td>
<td>VA - VB</td>
<td>IA - IB</td>
<td>-j * VC1m</td>
<td>Tab</td>
</tr>
<tr>
<td>BC</td>
<td>VB - VC</td>
<td>IB - IC</td>
<td>-j * VA1m</td>
<td>Tbc</td>
</tr>
<tr>
<td>CA</td>
<td>VC - VA</td>
<td>IC - IA</td>
<td>-j * VB1m</td>
<td>Tca</td>
</tr>
</tbody>
</table>

m: denotes memory voltage  
K = 1/3 (ZO/ZL - 1) ... residual current compensation factor

Each torque is a product with the following form:

\[ T = \text{Re} \left[ (Z_i^*I - V) \times VP^* \right] \]

In a microprocessor implementation, the computer calculates the six torque-like products and tests their signs. Positive products indicate impedances inside the expanded mho circle characteristics.

Positive-sequence memory voltage polarization provides expansion which maximizes coverage for high-resistance faults. Memory ensures that all elements operate reliably and securely for at least as long as the memory quantity lasts.

**Forming the Positive-Sequence Polarizing Voltage with Memory**

The block diagram in Figure 6 shows how a relay can determine positive-sequence memory voltage polarizing quantities from the three-phase voltages. The following description assumes a digital implementation.

The relay filters and samples each of the voltages VA, VB, and VC every 90 degrees (or four times per power cycle). A digital filter removes dc offset. The result is a phasor for each of the three voltages.

Next, the computer calculates the positive-sequence voltage, referred to phase A, from the voltage phasors. Calculations use the following equation:

\[ VA1 = \left[ VA + (a-1)VB + (a^2-1)VC \right]/3, \]

where the constant "a" has magnitude one and angle 120 degrees. The coefficients of VB and VC are arranged to shift the voltages by ±60 degrees and invert the result instead of rotating
the voltages by ±120 degrees. The 60 degree shifts provide a better transient response, than would 120 degree shifts because 60 degree shifts require shorter time delays.

For the memory filter, we designed a very simple infinite-impulse-response digital filter. Its equation acts on phasor quantities updated every quarter cycle (90 degrees):

\[ VA1M_k = \frac{1}{16} (VA1_k) - \frac{15}{16} (VA1M_{k-1}) \]

where \( VA1M \) is the output of the memory filter, and the index \( k \) counts in one-quarter cycle time-steps. The memory filter brings its output forward from two time-steps earlier (1/2 cycle earlier or 180 degrees), inverts it and scales it by \( 15/16 \). The inversion removes the 180 degree phase shift introduced by the half-cycle delay. Finally, the memory filter adds on \( 1/16 \) of the newest positive-sequence voltage measurement.

This filter has a time constant of four cycles, and provides very strong polarization for well over 20 cycles. An oscillogram near the end of this paper shows test results for a solid three-phase bus fault where the trip signal is maintained for over one-half second.

Table 2 shows an example of the input/output relationship for the filter for an initial voltage phasor (16, 16) which is suddenly lost. The phasor output decays slowly, with very little phase shift. A much larger magnitude signal would be used in a practical design; however, the choice of a phasor with components of 16 makes the filter behavior easy to see.

<table>
<thead>
<tr>
<th>VA1 INPUT</th>
<th>VA1M OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>-16</td>
<td>-16</td>
</tr>
<tr>
<td>-16</td>
<td>-16</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>-15</td>
</tr>
<tr>
<td>0</td>
<td>-14</td>
</tr>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>-13</td>
</tr>
<tr>
<td>0</td>
<td>-12</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>-11</td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

Steady-state input of (-16, 16)
Input switches to zero and memory output slowly decays.
The block diagram in Figure 6 completes the set of polarizing voltages by rotating VA1M by ±60 degrees and inverting the two results to obtain VB1 and VC1. As our table for the hho elements showed earlier, VA1M, VB1M, and VC1M polarize the AG, BG, and CG elements. The polarization voltages for the BC, CA, and AB elements are the same as those for the AG, BG, and CG elements rotated -90 degrees. Rotations back 90 degrees are simple:

$$(x, y) \rightarrow (y, -x).$$

For example, if $(x, y)$ is the phasor for polarizing the AG element, then $(y, -x)$ is the phasor for polarizing the BC element.

**FAULT TYPE SELECTION BY TORQUE COMPARISON**

**General Idea of Using Torque as a Fault-Type Discriminant**

Compared to self-polarized hho elements, expanded, dynamic hho elements are more likely to operate for unintended fault types. We need a way to determine the fault type, for reasons listed earlier, even when more than one hho element operates. Intuition might suggest that the strongest operating torque should be produced by the hho element designed to measure the given fault.

First, consider a set of self-polarized hho elements responding to an AG fault just in front of the relay. The polarizing voltage VAG is very small, while the operating quantity $Z_r*(IA + k*IR) - VA$ is quite large. In the limit as the fault approaches the point of voltage measurement, the torque tends to zero. If the source is strong, other hho elements tend to operate. These other elements have much stronger polarizing quantities. Some are very likely to produce more operating torque than the AG element. Therefore, torques from self-polarized elements are useless for fault-type discrimination.

Now consider what happens when all hho elements are polarized with positive-sequence memory voltage. All elements receive the same magnitude of polarization voltage, at the correct angles, under all fault and open-pole conditions. Now, for the AG fault near the bus, the AG torque is very strong, because the polarizing voltage is not collapsing. A quick review of the operating quantities for all six hho elements suggests that the AG element produces the maximum torque for this fault.

Thus, operating torque might be a useful fault-type discriminant.

In the microprocessor implementations, we compute a torque-like product as described earlier. The sign of the product is the operate/restrain result of the relay characteristic. What other information do the torque magnitudes contain? Can the magnitudes help determine fault type?

To answer these questions and evaluate designs, we modeled the general two-source system of Figure 7 using the computer program MathCad. MathCad manipulates equations written just as mathematicians write them, not the way other computer programming languages require.
We included the relay elements from Table 1 and observed the torques produced for thousands of system conditions and faults.

**Initial Evaluation of Torque as a Fault-Type Discriminant**

As a first experiment, we calculated torques for relay elements at one end of the system in Figure 7. Table 3 lists system impedances and shows torques produced by the six positive-sequence memory-polarized elements set to 200% of line impedance. These six elements could be the Zone 3 elements, which may be required to determine fault type as well as provide the third zone of protection. The torques are for AG, BC, and BCG faults at Bus S (m=0), line midpoint (m=0.5), and Bus R (m=1). The example assumes no power flow (power angle $\delta=0$) and zero fault resistance ($R_f=0$). Positive torques indicate elements which operated for the fault.

<table>
<thead>
<tr>
<th>Row</th>
<th>Fault Type</th>
<th>Fault Location (p.u.)</th>
<th>AG</th>
<th>BG</th>
<th>CG</th>
<th>AB</th>
<th>BC</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AG</td>
<td>0</td>
<td>67</td>
<td>-20</td>
<td>-20</td>
<td>28</td>
<td>-8.5</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>AG</td>
<td>.5</td>
<td>1.0</td>
<td>-7.9</td>
<td>-7.9</td>
<td>0.3</td>
<td>-8.5</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>AG</td>
<td>1</td>
<td>4.1</td>
<td>-5.2</td>
<td>-5.2</td>
<td>-1.1</td>
<td>-8.5</td>
<td>-1.1</td>
</tr>
<tr>
<td>4</td>
<td>BC</td>
<td>0</td>
<td>-4.9</td>
<td>28</td>
<td>28</td>
<td>11</td>
<td>68</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>BC</td>
<td>.5</td>
<td>-4.9</td>
<td>6.1</td>
<td>6.1</td>
<td>-2.1</td>
<td>17</td>
<td>-2.1</td>
</tr>
<tr>
<td>6</td>
<td>BC</td>
<td>1</td>
<td>-4.9</td>
<td>1.7</td>
<td>1.7</td>
<td>-4.7</td>
<td>6.8</td>
<td>-4.7</td>
</tr>
<tr>
<td>7</td>
<td>BCG</td>
<td>0</td>
<td>-34</td>
<td>53</td>
<td>53</td>
<td>28</td>
<td>68</td>
<td>28</td>
</tr>
<tr>
<td>8</td>
<td>BCG</td>
<td>.5</td>
<td>-9.5</td>
<td>10.0</td>
<td>10.0</td>
<td>1.2</td>
<td>7.0</td>
<td>1.2</td>
</tr>
<tr>
<td>9</td>
<td>BCG</td>
<td>1</td>
<td>-54</td>
<td>4.0</td>
<td>4.0</td>
<td>-1.1</td>
<td>6.8</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ZS1=2</th>
<th>ZL1=8</th>
<th>ZR1=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZS0=2</td>
<td>ZL0=24</td>
<td>ZR0=2</td>
</tr>
<tr>
<td>$\delta=0$</td>
<td>$R_f=0$</td>
<td>Reach=2 p.u.</td>
</tr>
</tbody>
</table>

The maximum operating torque for each fault listed in Table 3 is always produced by the mho element designed to cover its particular fault type.

Row 1 of Table 3 shows the six torques for an AG fault at Bus S. Three elements operate: AG, AB, and CA; however, the AG element, corresponding to the AG fault type, produces the maximum torque of 67.
The mid-line fault in row 2 of Table 3 operates the AG element and barely operates the AB and CA elements. Again, the maximum torque belongs to the AG element, which we associate with the fault type.

The row-3 fault is at the line end. Only the AG element operates.

The BC bus fault (row 4) operates every element except the AG element, yet the BC element produces maximum torque.

The three solid BCG faults in rows 7, 8, and 9 should be measured by the BC element. A maximum-torque strategy again selects the proper element.

In conclusion, torque appears to be a useful fault-type discriminant. Therefore, we studied many systems, system conditions, and fault conditions to determine fault-type selecting capabilities and limitations of positive-sequence memory-polarized mho elements. The next sections cover practical considerations and some of the simulation results.

PRACTICAL CONSIDERATIONS AND SCHEMES

Studies and analysis showed that mho element operating torque can be used with a fault-type identification table to discriminate between fault types over a very broad range of system configurations. We can optimize scheme performance through torque analysis, choosing between selectivity and sensitivity as desired. Examples follow:

1. Under some system conditions, as the resistance of an AG fault increases, the CG element torque can exceed the AG element torque. We can now choose between sensitivity and selectivity. Three-pole schemes can use the maximum-torque element for maximum sensitivity; single-pole schemes must identify the faulted phase. A lookup table relating the AG-CG combination to the AG fault type satisfies the single-pole requirement.

2. Under strong-source system conditions, as the resistance of a close-in AG fault increases, the AB element torque might exceed the AG element torque. Again, we can choose between sensitivity in a three-pole-trip scheme and selectivity in a single-pole-trip scheme. The $\text{Tab} > \text{Tag}$ condition could cause a three-pole trip for an AG fault given certain systems and fault resistances. However, this torque condition is associated with a unique combination of picked-up relay elements: AG, CG, AB. A lookup table can relate this combination to the AG fault condition to provide the desired single-pole trip.

3. For phase-to-phase faults, tripping is three-pole, and selecting the maximum-torque element yields the most sensitivity.

4. For double-line-to-ground faults, torque is a good discriminant in selecting the correct phase-to-phase element over a ground element. Torque discrimination can block the ground elements which tend to overreach for double-line-to-ground faults with some resistance to ground.
Figure 8 shows the block diagram for a practical scheme. Every quarter cycle, the Zone 3 mho elements update and present their operate/restrain states to a fault identification table. The table identifies the fault type directly, if possible. If the combination of picked-up elements does not directly indicate fault type, a subsequent process compares selected element torques to determine fault type.

SIMULATION AND TEST STUDIES

The source-to-line impedance ratio strongly influences the unwanted operation of relay elements. Strong-source systems tend to operate more relay elements for a given fault type than weak-source systems. Therefore, phase selection and fault-type identification are much more difficult in strong-source systems. We chose system conditions which pose the most difficult challenges to phase selection and fault-type identification.

Selectivity Between Close-In Phase-to-Phase and Phase-to-Ground Faults

A close-in ground fault in front of a strong source operates one or both related phase elements, as well as the intended ground element. Figure 9 graphs torque ratios $\frac{\text{Tab}}{\text{Tag}}$ as a function of fault resistance for an AG fault at the bus. Figure 9a shows the results for a strong source behind Bus S, while Figure 9b shows the results for a weak source behind Bus S. For the weaker system ($ZS1=ZS0=6$), the torque ratio remains less than 0.5 for all values of $Rf$. Thus, the torque ratio is an excellent discriminant.

For the strong-source system shown in Figure 9b, torque discriminates properly between phase and ground faults up to a fault resistance of approximately three ohms. Torque discrimination above three ohms is not important, since the picked-up mho element combination directly indicates the AG fault type using the fault-identification lookup table shown in Figure 8.

Load flow has very little effect on selectivity by torque. Therefore, the figures show torque ratios for a zero power angle only.

Phase and Ground Element Selectivity for Double-Line-to-Ground Faults

Zone 1 must not operate for a fault beyond the next bus. A double-line-to-ground fault (LLG) with resistance to ground tends to cause the ground element associated with the leading phase to overreach for certain values of fault resistance. The relay scheme must detect the LLG fault type and block or restrict Zone 1 ground distance elements.

Figure 10 shows the effectiveness of torque in this application. The mho elements reach 200% of the line, and the BCG fault is at the next bus, or 100% of the line.
The torque ratio $T_{bc}/T_{bg}$ remains greater than one, ensuring that the BC element is favored, and the ratio can be used to block the BG element. Although the power flow as expressed by the power angle $\delta$ has considerable effect on the torque ratio, it does not interfere with discrimination between BCG and BG faults.

Other tests demonstrate that the discriminant becomes even stronger as the relay reach increases.

**Maximum-Torque Selection Yields Maximum Sensitivity**

As the resistance in a BC fault increases, it is possible for the torque produced by the CA element to exceed that produced by the BC element. As the fault resistance increase continues, the BC element may drop out, leaving only the CA element picked up.

Figure 11 plots the torques for phase-to-phase elements set to 90% and 200% of the line as functions of fault resistance. A BC fault is at Bus S, the relay location. The 200% setting corresponds to Zone 3, which might be used for fault-type selection. The 90% setting corresponds to Zone 1.

For $R_f=0$, all three phase-to-phase elements operate for reach settings of 90% and 200%. The AB element torques quickly drop off, leaving only BC and CA picked up.

The BC and CA torques are equal at a fault resistance of 4.55 ohms for both the Zone 3 and Zone 1 elements.

Using a maximum-torque scheme, the relay selects BC elements for Zones 1 and 2 for fault resistances below 4.55 ohms, and selects the CA element for fault resistances above 4.55 ohms. In doing so, the relay operates the more sensitive element in the inner zones, even though the element selected does not correspond to the fault type. The advantage of this method is greatly expanded resistance coverage. The disadvantage is that the relay targets CA for a BC fault if the fault resistance is very high. However, the event reports retained by the digital relays indicate the actual voltages and currents, from which an engineer can deduce the fault type.

**Open-Pole Security**

In single-pole tripping schemes, polarization must ensure that the relay elements for uninvolved phases do not operate during the single-pole-open condition.

Positive-sequence memory voltage polarization achieves that requirement, and makes the expanded mho elements very stable during open-pole intervals. The stability stems from the virtually constant phase relationship maintained by the polarizing quantities for all elements, for all disturbances (faults, open poles, etc.).

As an example, consider the following system of the form shown in Figure 7:

\[
\begin{align*}
ZS1 &= 2 \\
ZL1 &= 8 \\
ZR1 &= 2 \\
ZS0 &= 6 \\
ZL0 &= 24 \\
ZR0 &= 6
\end{align*}
\]
Loads up to $\delta = \pm 60$ degrees do not enter mho characteristics set to reach 32 ohms (400% of the line) under normal balanced operation.

If pole A opens due to an AG fault, then the elements remain secure for power angles in the range of $-59 < \delta < 52$ degrees, as long as phase-A overcurrent elements block the operation of the AB and CA mho elements.

**Memory Polarization Effectiveness**

To demonstrate the effectiveness of the memory filter described earlier, we applied a zero-voltage three-phase fault to a digital relay with the digital memory filter. Figure 12 shows the trip signal from the relay, along with one current and one voltage. The relay maintained its trip signal for 600 ms after the total voltage collapse. Dropout of the trip signal occurred when logic associated with the memory filter declared that the polarization voltage had decayed to the point of being incapable of providing reliable phase information.

**CONCLUSIONS**

Positive-sequence memory voltage polarizes mho elements soundly for all faults. The resulting expanded mho characteristics cover resistive faults better. A microprocessor-based relay easily forms the polarization quantities.

Torques generated by the expanded mho elements help identify fault types and provide design flexibility for single-pole and three-pole trip schemes.

Torque comparison is a powerful new tool for discriminating between ground, phase, and double-line-to-ground faults under a wide range of system and fault conditions.

**REFERENCES**


Figure 1: Development of Self-Polarized Mho Relay Characteristics

Figure 2: Development of Expanded Mho Characteristic

Figure 3: Phase-to-Phase Element Response for Three-Phase Forward Fault
Figure 4: BC Element Response for BC Forward Fault

Figure 5: Phase-Ground Element Response for Phase-Ground Fault

Figure 6: Positive-Sequence Polarizing Voltage Block Diagram
Figure 7: Model for Evaluating Torque Comparisons

Figure 8: Block Diagram for a Practical Scheme Employing Torque Comparison
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Figure 11: BC Fault Torques in AB, BC, and CA Elements Set to 0.9 and 2.0 p.u. of Line Length for BC Fault at Bus S
Figure 12: Memory Polarization Performance for Forward Fault