Using Thermal Limit Curves to Define Thermal Models of Induction Motors

Stanley E. Zocholl and Gabriel Benmouyal
Schweitzer Engineering Laboratories, Inc.

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I. INTRODUCTION

IEEE Standard C37.96-2000, *Guide for AC Motor Protection* [1], recommends the use of overcurrent relays for overload and locked rotor protection. In these applications, setting the overcurrent inverse time-current characteristic to coordinate with the motor thermal limit curves provides protection. Because of the familiar use of overcurrent protection, little attention is paid to the nature of motor thermal limit curves and their relation to winding temperature in an induction motor. Yet, the limit curves are the characteristics of thermal models that enable microprocessor relays to continuously calculate and monitor motor temperature in real time. This paper uses induction motor thermal limit curves to define the thermal model and correlate motor current with temperature. The paper employs MATLAB simulations to compare the dynamics of the thermal model with that of an overcurrent implementation during cyclic overloads.

II. THERMAL LIMIT CURVES

The thermal limit curves for a 400-horsepower, 3600-rotations-per-minute, 440-volt induction motor are shown in Fig. 1. By definition, the curves give the time for current exceeding the service factor (SF) to raise the initial load temperature to an overload temperature that requires the motor to be disconnected. The SF in this case is 1.15. The manufacturer has specified the initial temperature for a set of "hot" and "cold" overload and locked rotor curves.
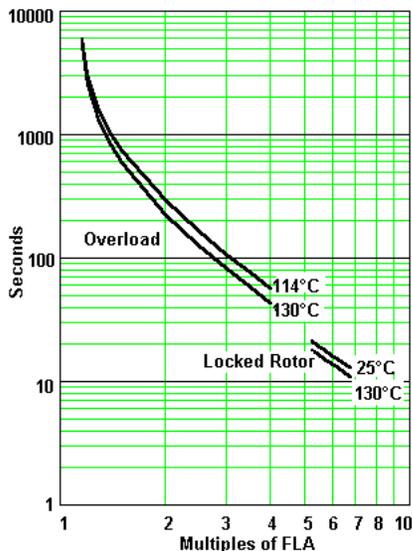

Fig. 1. Hot and Cold Limit Curves for a 400-Horsepower Motor, SF = 1.15

The initial motor temperature values are marked on each curve. The curves are presented in accordance with guidelines in IEEE Standard 620-1996, *Guide for the Presentation of Thermal Limit Curves for Squirrel Cage Induction Machines* [2]. Otherwise, the guide gives no information as to how the curves are constructed. However, we can conclude that each curve is a plot of a specific limiting temperature.

III. DEFINING THE THERMAL MODELS

The basic equations found in thermal protection modeling are derived in the Annex. Equation (A21) gives us the form for overload thermal limit curves plotted in Fig. 1. The equations for the overload curves of Fig. 1 have the form:

$$t_{H-CURVE} = T_{th} \ln\left(\frac{I^2 - I_H^2}{I^2 - I_{SF}^2}\right)$$

$$t_{C-CURVE} = T_{th} \ln\left(\frac{I^2 - I_C^2}{I^2 - I_{SF}^2}\right) \tag{1}$$

where:

T_{th} is the thermal time constant.

I is the motor current in per unit (pu) of full load.

I_{SF} is the current at the service factor.

I_H is the current that raised the temperature to 130°C.

I_C is the current that raised the temperature to 114°C.

If the curves obey a first order thermal process, we will be able to choose T_{th}, I_H, and I_C so that the equations fit the curves. Also, I_H and I_C must be in the ratio of the initial temperatures above ambient. A unique solution is obtained under the necessary constraint that:

$$\frac{I_H^2}{I_C^2} = \left(\frac{130-25}{114-25}\right) = 1.179 \tag{2}$$

$$I_C^2 = \frac{I_H^2}{1.179} \tag{3}$$

The fit is obtained by satisfying simultaneous equations (4) where the equations have been solved for the service factor SF^2. The curve fitting procedure is as follows:

1. Choose a current and read the corresponding time points from the hot (130°C) and the cold (114°C) overload curves in Fig. 1. Enter the current and time values in (4). For example, at 2 per unit current, the hot and cold times are $t_{H-CURVE} = 223$ seconds and $t_{C-CURVE} = 279$ seconds, respectively.

2. Choose the T_{th} and I_H so that equations (4) equal the service factor.
3. Cut and try to refine the values. For example, when $T_{th} = 1370$ seconds and $I_H = 0.92$ per unit, both equations equal $1.15^2 = 1.322$.
4. Check the result at other points on the curve. Curve values are shown in Table I.

$$I^2\left(1 - e^{-\frac{t_{H-CURVE}}{T_{th}}}\right) + I_H^2 \cdot e^{-\frac{t_{H-CURVE}}{T_{th}}} = 1.15^2$$

$$I^2\left(1 - e^{-\frac{t_{C-CURVE}}{T_{th}}}\right) + \frac{I_H^2}{1.179} e^{-\frac{t_{C-CURVE}}{T_{th}}} = 1.15^2 \tag{4}$$

TABLE I
THERMAL LIMIT CURVE CHECKPOINTS

I (pu)	$t_{H-CURVE}$ (s)	$t_{C-CURVE}$ (s)	T_{th} (s)	I_H^2 (pu)	I_C^2 (pu)	I_{SF} (pu)
2.0	223	279	1370	0.846	0.717	1.15
2.5	126	158	1370	0.846	0.717	1.15
3.0	82	104	1370	0.846	0.717	1.15

When the unique values for T_{th}, I_H, and I_C are used, (5) can be used to calculate any curve point with precision:

$$t_H = 1370 \cdot \ln\left(\frac{I^2 - 0.846}{I^2 - (1.15)^2}\right)$$

$$t_C = 1370 \cdot \ln\left(\frac{I^2 - 0.717}{I^2 - (1.15)^2}\right) \tag{5}$$

The equations in (5) are the solutions of a first order differential equation that we will use to implement the thermal model:

$$I^2 R_T = R_T C_T \cdot \frac{d\theta}{dt} + \theta \tag{6}$$

where:
  $R_T C_T$ is the thermal time constant = 1370 seconds
  θ is temperature at t = 0
  R_T is 1

The time-discrete form of (6) can be written as:

$$I^2 R_T = R_T C_T \cdot \frac{\theta_n - \theta_{n-1}}{\Delta t} + \theta_{n-1}$$

Solving for θ_n gives the time-discrete form of the differential equation:

$$\theta_n = \frac{I^2}{C_T} \Delta t + \left(1 - \frac{\Delta t}{R_T C_T}\right) \cdot \theta_{n-1} \tag{7}$$

Equation (7) is the algorithm that enables a microprocessor relay to continuously calculate motor temperature. The temperature is then compared to predetermined trip and alarm thresholds to provide thermal protection under any condition of operation.

A. Thermal Versus Overcurrent Model

It is instructive to compare response of the thermal model to an overcurrent model of a thermal limit curve. The overcurrent model is implemented by integrating the reciprocal of the hot thermal limit curve as specified in Equation 3 of the IEEE Standard C37.112-1996 [3]. The incremental equations for this process are:

$$t_H = 1370 \cdot \ln\left(\frac{I^2 - .846}{I^2 - (1.15)^2}\right) \tag{8}$$

For I > 1.15:

$$\theta_n = \theta_{n-1} + \frac{\Delta t}{t_H} \tag{9}$$

For I ≤ 1.15:

$$\theta_n = \left(1 - \frac{\Delta t}{1370}\right) \cdot \theta_{n-1} \tag{10}$$

Equation (9) is used to calculate the response of the overcurrent relay above the pickup current. Equation (8) is the time-current characteristic. θ_n and θ_{n-1} are consecutive samples displaced by one time increment. Below pickup, the overcurrent relay resets exponentially using the thermal time constant to emulate the cooling of the motor.

Fig. 2 shows the response of both models to a current below pickup. Whereas the overcurrent model has no response, the thermal model calculates the temperature that rises exponentially toward the steady-state temperature $\theta = 0.846$. In Fig. 3, both models are subjected to a cyclic overload with the motor initially at 0.846 per unit of thermal capacity.
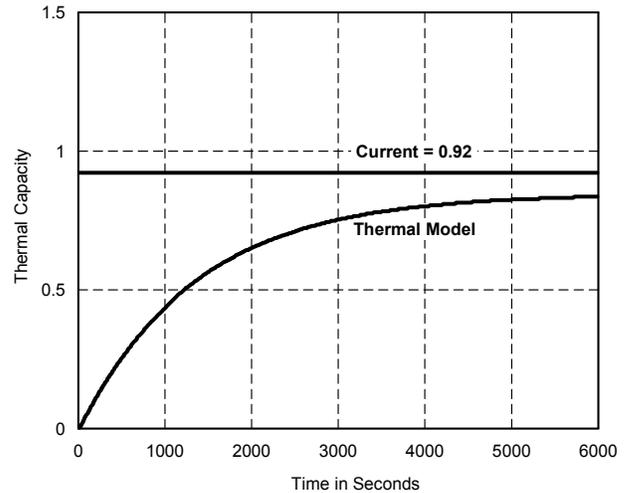

Fig. 2. Response of Models to a Current Less Than the Service Factor (I = 0.92)
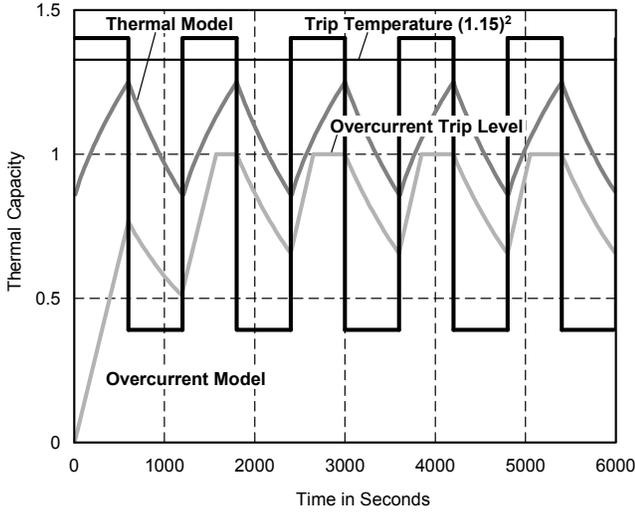
Fig. 3.   Response of the Models to a Cyclic Overload Current

The cyclic current in Fig. 2 alternates between 1.4 and 0.4 per unit current every 10 minutes. Note that the average of the currents squared and the rms current is:

$$I_{2aver}^2 = \frac{I_{high}^2 + I_{low}^2}{2} = \frac{1.4^2 + .4^2}{2} = 1.06 \tag{11}$$
$$I_{RMS} = \sqrt{1.06} = 1.03$$

The cyclic current is not an overload that raises the temperature to the trip value. Fig. 3 shows the cyclic temperature response of the thermal model reaches a 1.06 average, or 80 percent of the trip value.

The overcurrent relay model does not measure temperature and trips because it cannot account for thermal history.

Fig. 4 shows that the overcurrent and the thermal model produce the same trip time when the motor is initially at the temperature specified for the hot thermal limit curve. Fig. 5 shows the trip times when the motor is initially at the temperature specified for the cold thermal limit curve. The thermal model trips at the limiting temperature while the overcurrent trips at the hot curve time, independent of the initial temperature of the motor.
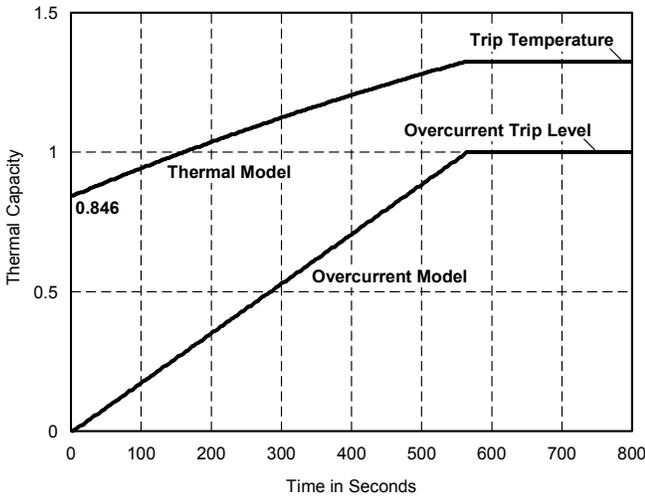

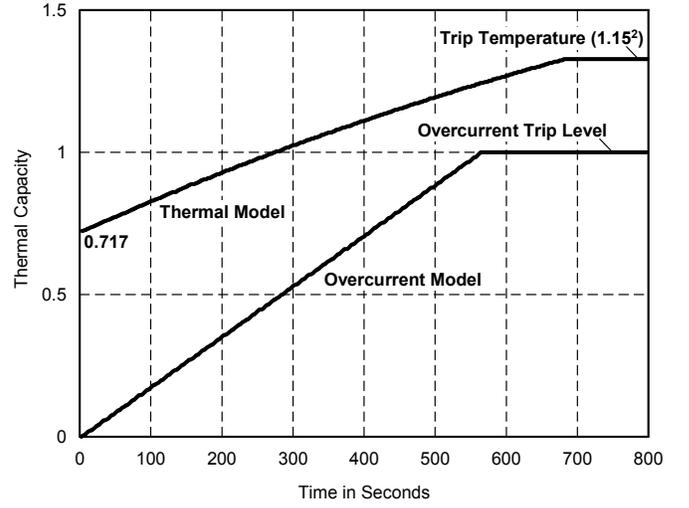Fig. 4.   Thermal and Overcurrent Model Response With Initial Temperature = 0.846

Fig. 5.   Thermal and Overcurrent Model Response With Initial Temperature = 0.717

B. Correlation of Current With Temperature

The initial temperatures for the "hot" and "cold" overload thermal limit curves are given as 130°C and 114°C, respectively. Also, the equations in (5) give the squares of the initial currents as 0.846 and 0.717 per unit, which can be read as per unit watts in the thermal model. Since temperature is proportional to current squared, we can write:

$$\frac{\theta}{I^2} = \frac{(130-25)-(114-25)}{0.846-0.717} \tag{12}$$

Where θ is the motor temperature rise above ambient and I is current in per unit, we can then write:

$$\theta = \frac{(130-25)-(114-25)}{0.846-0.717}I^2 + 25 = 124.031 \cdot I^2 + 25 \tag{13}$$

Where θ the steady-state temperature in degrees centigrade for a given per unit current, tabulated values are as shown in Table II.

TABLE II
CORRELATION OF CURRENT TO TEMPERATURE

Motor Current (Per Unit)	Temperature
1.15 (Service Factor)	189°C
0.92	130°C
0.85	114°C
0.0	25°C

IV. CONCLUSIONS

1. In this paper, we introduced the basic principles and mathematical equations found in thermal protection modeling.
2. We showed that the first order thermal equation fits the thermal limit curves of a 400-horsepower, 3600-rotations-per-minute, 440-volt motor. We then used the hot and cold thermal model equations and the initial temperature data to derive the time constant of the thermal model.

3. We showed the hot thermal limit equation implemented as an overcurrent model. Using MATLAB simulations, we compared the dynamic responses of the overcurrent and thermal models.
4. The simulations showed that the thermal model determines the true temperature caused by cyclic overload current, while the overcurrent model trips for cyclic currents that do not overheat the motor.
5. Finally, we used the initial current for the hot and cold equations and the initial temperature data to correlate the motor current with temperature in degrees centigrade.

V. ANNEX: BASICS OF THERMAL MODELING

This annex introduces the basic principles and mathematical equations found in thermal protection modeling by resolving a simple first order thermal process: the heating by a resistor of a vessel containing 1 liter of water.

A. A Simple Thermal System

Let us assume a vessel contains 1 liter (or 1 kilogram) of water. The water is heated by a source of energy V. θ_A is the ambient temperature surrounding the vessel, and θ_w is the water temperature. We do not want the water to go beyond the boiling point, or 100 degrees centigrade. This temperature is defined as the maximum or hot-spot temperature θ_{max}, and above this point, the voltage source will be disconnected by some protective device.
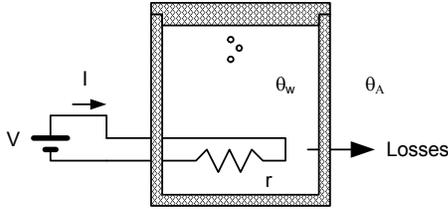

Fig. A1. A Simple Thermal System

Let us define θ as the temperature of the water above ambient or:

$$\theta = \theta_w - \theta_A \qquad (A1)$$

The rate of increase of the water temperature is provided by the equation expressing the thermal equilibrium.

Power Supplied to the Water – Losses

$$= C_s m \frac{d\theta_w}{dt} = C_s m \frac{d\theta}{dt} \qquad (A2)$$

In this equation, C_s is the specific heat of the water. It corresponds to the amount of energy that has to be supplied to 1 kilogram of water to raise its temperature by 1 degree centigrade. Its value is 4.19E3 joules kg/°C. m, in kilograms, is the mass of the water. The losses or the quantity of heat transferred by the vessel to the surrounding environment can be expressed as:

$$\text{Losses} = \frac{\theta_w - \theta}{R} \qquad (A3)$$

Equation (A2) can be otherwise expressed as:

$$I^2 r - \frac{\theta}{R} = C_s m \frac{d\theta}{dt} \qquad (A4)$$

or:

$$I^2 r \bullet R = C_s m \bullet R \frac{d\theta}{dt} + \theta \qquad (A5)$$

The mass m multiplied by the specific heat C_s is known as C, the thermal capacity of the system with units of joules/°C. It represents the amount of energy in joules required to raise the system temperature by 1 degree centigrade.

The product of the thermal resistivity R and the thermal capacity C has a unit of seconds and represents the thermal time constant T_{th} of the system:

$$T_{th} = R \bullet m \bullet C_s \qquad (A6)$$

The solution in the time domain for the temperature as a function of time and current is:

$$\theta = I^2 r \bullet R \left(1 - e^{-\frac{t}{R \bullet m \bullet C_s}} \right) \qquad (A7)$$

Remembering that θ is the temperature above ambient, we obtain for the expression of the water temperature:

$$\theta_W(t) = I^2 r \bullet R \left(1 - e^{-\frac{t}{T_{th}}} \right) + \theta_A \qquad (A8)$$

Let us assume the system has some rated operating current I_{op} (which is otherwise called the load current in some applications). Then the water operating temperature is given by:

$$\theta_{op} = I_{op}^2 r \bullet R + \theta_A \qquad (A9)$$

Whatever the current supplied to the vessel, there will always be an increase in the water temperature. The final water temperature for a constant current I is given by:

$$\theta_W(0 \to \infty) = I^2 r \bullet R + \theta_A \qquad (A10)$$

Equation (A5) is a first order differential equation and has an electrical equivalent: an RC circuit supplied by a current source. The power supplied to the water in the thermal process is equivalent to the current source supplying the RC circuit. The temperature in the thermal process is equivalent to the voltage across the capacitor in the RC circuit. The equivalence between the two systems is shown in Fig. A2.

When supplied by a current step function, the temperature or voltage time response is shown in the same figure.
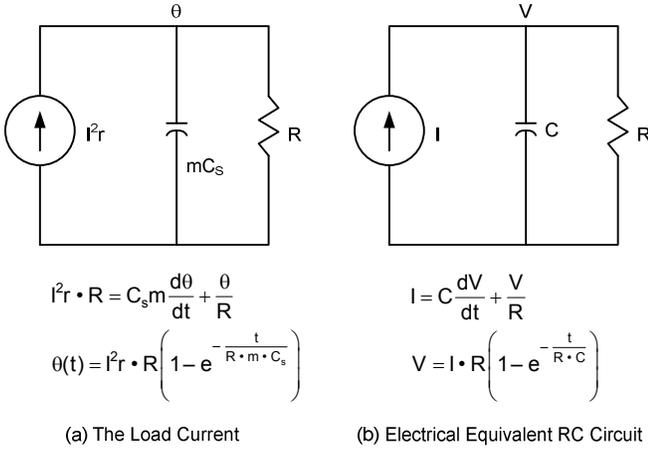
$$I^2r \cdot R = C_s m \frac{d\theta}{dt} + \frac{\theta}{R}$$

$$\theta(t) = I^2 r \cdot R \left(1 - e^{-\frac{t}{R \cdot m \cdot C_s}} \right)$$

(a) The Load Current

$$I = C \frac{dV}{dt} + \frac{V}{R}$$

$$V = I \cdot R \left(1 - e^{-\frac{t}{R \cdot C}} \right)$$

(b) Electrical Equivalent RC Circuit

Fig. A2.　Equivalence Between a Thermal System and a Parallel RC Circuit

B. The Equivalent Time-Current Curve

If the water temperature must not go beyond a maximum temperature θ_{max}, then the equation with the time as a variable is:

$$\theta_{max} = I^2 r \cdot R \left(1 - e^{-\frac{t}{T_{th}}} \right) + \theta_A \qquad (A11)$$

Solving for t gives the time-current equation:

$$t = T_{th} \ln \left(\frac{I^2 r \cdot R}{I^2 r \cdot R - (\theta_{max} - \theta_A)} \right) \qquad (A12)$$

Let us define the current I_{max} as the maximum current that can be supplied to the heating resistor without the water reaching maximum temperature as time goes to infinity. This maximum current would have to satisfy (A10) as in:

$$\theta_{max} = I_{max} r \cdot R + \theta_A \qquad (A13)$$

or:

$$I_{max}^2 r \cdot R = \theta_{max} - \theta_A \qquad (A14)$$

Substituting $\theta_{max} - \theta_A$ in Equation (A12) gives the equation:

$$t = T_{th} \ln \left(\frac{I^2 r \cdot R}{I^2 r \cdot R - I_{max}^2 r \cdot R} \right) = T_{th} \ln \left(\frac{I^2}{I^2 - I_{max}^2} \right) \quad (A15)$$

In (A15), we have finally expressed the time to reach the hot-spot temperature as a function of the current. Equation (A15) is also remarkable because we have removed all the temperature constants and replaced them with the maximum current I_{max}.

Based on (A11) and (A15), the electrical equivalent circuit for the thermal process can be represented by the RC circuit, as shown in Fig. A2.

(a) Maximum Threshold as Function of Temperatures

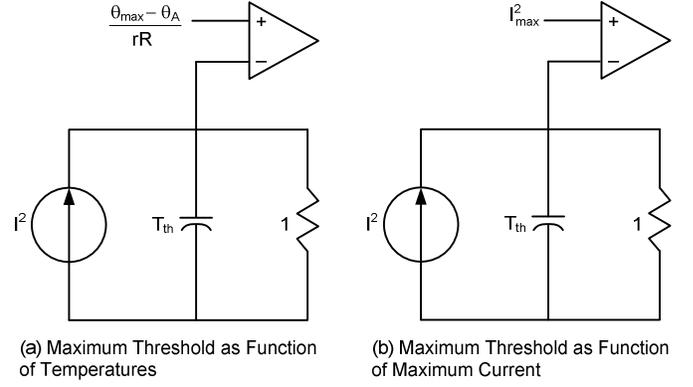(b) Maximum Threshold as Function of Maximum Current

Fig. A3.　Electrical RC Equivalent of Thermal Process

It should be noted that (A15) has no solution unless:

$$I > I_{max} \qquad (A16)$$

Any current less than I_{max} will raise the water temperature to a steady temperature given by (A7). This temperature will be represented by a capacitor voltage in the equivalent circuit of Fig. A3(b).

In (A15), the time to maximum temperature is expressed implicitly with reference to the ambient temperature or with the initial current equal to zero. Let us assume we want to develop an expression for the time to hot-spot temperature when the steady-state current is the operating current I_{op}. In (A17), the time to maximum temperature starts with the temperature at ambient (or with the current supplied at zero value). With the newer equation, the time to maximum temperature starts with the temperature at operating or the current at the load current.

The time to reach the maximum temperature for some current I from the operating current (or temperature) is equal to the time to reach the maximum current from ambient with the same current minus the time to reach the operating temperature from ambient with the same current.

We can compute θ_{op} from (A9):

$$\theta_{op} = I_{op}^2 r \cdot R + \theta_A \qquad (A17)$$

The time to reach the operating temperature t_{op} from ambient for a current I can be computed from (A8) as:

$$\theta_{op} = I^2 r \cdot R \left(1 - e^{-\frac{t_{op}}{T_{th}}} \right) + \theta_A \qquad (A18)$$

from which we get t_{op} as:

$$t_{op} = T_{th} \ln \left(\frac{I^2 r \cdot R}{I^2 r \cdot R - (\theta_{op} - \theta_A)} \right) \qquad (A19)$$

Replacing θ_{op} by its value, we get:

$$t = T_{th} \ln\left(\frac{I^2 r \cdot R}{I^2 r \cdot R - I_{op}^2 r \cdot R}\right) = T_{th} \ln\left(\frac{I^2}{I^2 - I_{op}^2}\right) \quad (A20)$$

Finally, the time from operating current or temperature is provided by:

$$t = T_{th} \ln\left(\frac{I^2}{I^2 - I_{max}^2}\right) - T_{th} \ln\left(\frac{I^2}{I^2 - I_{op}^2}\right) = \\ T_{th} \ln\left(\frac{I^2 - I_{op}^2}{I^2 - I_{max}^2}\right) \quad (A21)$$

This latter expression provides the time to reach the hot-spot temperature for a current I when starting from the operating current or operating temperature.

Normalizing the current with respect to the operating current, we get finally:

$$t = T_{th} \ln\left(\frac{I_{pu}^2 - 1}{I_{pu}^2 - SF^2}\right) \quad (A22)$$

Where SF, or the service factor, is defined as the ratio of the maximum current over the operating current:

$$SF = \frac{I_{max}}{I_{op}} \quad (A23)$$

The electrical RC equivalent of the thermal process represented by (A22) is shown in Fig. A4.

Note that the models in Fig. A4 are identical to the models in Fig. A3 except that the current is expressed in per unit. In (A21), time is measured starting from an initial operating temperature. This condition is represented in the model of Fig. A4 by a voltage across the capacitor equivalent to the initial operating temperature.
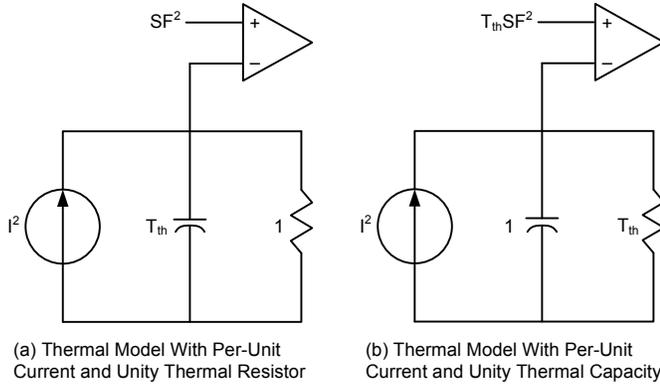

(a) Thermal Model With Per-Unit Current and Unity Thermal Resistor

(b) Thermal Model With Per-Unit Current and Unity Thermal Capacity

Fig. A 4.   Electrical RC Equivalent of Thermal Process

Starting from (A11), we can define the following invariant quantity:

$$\theta - \theta_A = I^2 r \cdot R\left(1 - e^{-\frac{t}{T_{th}}}\right) \quad (A24)$$

Equation (A24) indicates that for any pair of points (I_0, T_0) and (I_1, T_1) on the time-current curve, we have the following identity:

$$\text{Constant} = I_1^2 r \cdot R\left(1 - e^{-\frac{t_1}{T_{th}}}\right) = I_0^2 r \cdot R\left(1 - e^{-\frac{t_0}{T_{th}}}\right) \quad (A25)$$

Equation (A25) can otherwise be expressed as:

$$\frac{\left(1 - e^{-\frac{t_1}{T_{th}}}\right)}{\left(1 - e^{-\frac{t_0}{T_{th}}}\right)} = \frac{I_0}{I_1} \quad (A26)$$

Relations (A25) and (A26) determine if a given time-current curve corresponds to a first order thermal model.

VI. REFERENCES

[1] IEEE Std C37.96-2000, *Guide for AC Motor Protection*.

[2] IEEE Std 620-1996, *Guide for the Presentation of Thermal Limit Curves for Squirrel Cage Induction Machines*.

[3] IEEE Std C37.112-1996, *Standard Inverse-Time Characteristics Equations for Overcurrent Relays*, pp. 2–4.

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VII. BIOGRAPHIES

Stanley E. Zocholl has a B.S. and M.S. in Electrical Engineering from Drexel University. He is an IEEE Life Fellow and a member of the Power Engineering Society and the Industrial Application Society. He is also a member of the Power System Relaying Committee and past chair of the Relay Input Sources Subcommittee. He joined Schweitzer Engineering Laboratories, Inc. in 1991 in the position of Distinguished Engineer. He was with ABB Power T&D Company Allentown (formerly ITE, Gould, BBC) since 1947, where he held various engineering positions, including Director of Protection Technology. His biography appears in Who's Who in America. He holds over a dozen patents associated with power system protection using solid state and microprocessor technology and is the author of numerous IEEE and Protective Relay Conference papers. He received the Best Paper Award of the 1988 Petroleum and Chemical Industry Conference and the Power System Relaying Committee's Distinguished Service Award in 1991.

Gabriel Benmouyal received his B.A.Sc. in Electrical Engineering and his M.A.Sc. in Control Engineering from Ecole Polytechnique, Université de Montréal, Canada, in 1968 and 1970, respectively. In 1969, he joined Hydro-Québec as an instrumentation and control specialist. He worked on different projects in the field of substation control systems and dispatching centers. In 1978, he joined IREQ, where his main field of activity was the application of microprocessors and digital techniques to substation and generating station control and protection systems. In 1997, he joined Schweitzer Engineering Laboratories, Inc. in the position of Research Engineer. He is a registered professional engineer in the Province of Québec, is an IEEE member, and has served on the Power System Relaying Committee of IEEE since May 1989.