Implementing a Line Thermal Protection Element Using Programmable Logic

Gabriel Benmouyal, Michael B. Bryson, and Marc Palmer
Schweitzer Engineering Laboratories, Inc.

Presented at the
30th Annual Western Protective Relay Conference
Spokane, Washington
October 21–23, 2003
IMPLEMENTING A LINE THERMAL PROTECTION ELEMENT USING PROGRAMMABLE LOGIC

Gabriel Benmouyal
Schweitzer Engineering Laboratories
Longueuil, PQ CANADA

Michael B. Bryson
Schweitzer Engineering Laboratories
Pullman, WA USA

Marc Palmer
Schweitzer Engineering Laboratories
Llam, Christchurch NEW ZEALAND

ABSTRACT

This paper reviews the basic mathematical and physical principles of line thermal protection and presents the implementation of this element into a transmission line relay with advanced programmable logic capabilities. This type of protection is particularly useful when the load current carried by the line is close to the conductor ampacity limit.

Keywords: cable ampacity, protective relays, thermal element, thermal protection, transmission line thermal protection.

INTRODUCTION

Line thermal protection consists of monitoring the conductor temperature and generating a trip signal when the temperature becomes greater than the conductor maximum allowed temperature. The temperature is not measured directly but is emulated using a first-order thermal model. The conductor temperature is determined by establishing the heat-balance equation (heat input minus heat losses) of a conductor section 1000 feet in length. It is assumed that this section of the line is the hottest and it has been exposed to the maximum solar radiation. Heat input to the conductor is mainly due to heat dissipated in the conductor resistance and to solar heat gain. Heat losses are mainly due to convection and radiation losses. All of these principles and procedures have been standardized in the document IEEE Std 738-1993 “IEEE Standard for Calculating the Current-Temperature Relationship of Bare Overhead Conductors” [1] and the basic physical equations are summarized in [2].

The absence of proper line thermal protection could lead to catastrophic events. Thermally overloaded lines will develop excessive sag. The conductors’ sag in turn will increase the risks of faults from contact with nearby objects. Early reports indicated that this very scenario could have been one of the root causes that led to the North American blackout of August 2003.

This paper describes the implementation of the heat-balance equation as a first-order thermal model and the temperature rise equations using user-friendly programmable logic with mathematical capabilities in addition to conventional Boolean operations.

THE THERMAL MODEL OF A LINE CONDUCTOR

The mathematical and physical background of first-order thermal models is thoroughly explained in the Annex of [3].

Refer to Table 1 for the definition of model variables.
### Table 1  First-Order Thermal Model Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>heat power input to the conductor</td>
<td>kW/kft</td>
</tr>
<tr>
<td>THC</td>
<td>conductor heat thermal capacity</td>
<td>kJ/°C kft</td>
</tr>
<tr>
<td>TRA</td>
<td>thermal resistance to ambient</td>
<td>°C kft/kw</td>
</tr>
<tr>
<td>TC</td>
<td>estimated conductor temperature</td>
<td>°C</td>
</tr>
<tr>
<td>TA</td>
<td>ambient temperature</td>
<td>°C</td>
</tr>
<tr>
<td>TI</td>
<td>conductor initial temperature</td>
<td>°C</td>
</tr>
<tr>
<td>I</td>
<td>conductor current</td>
<td>A RMS</td>
</tr>
<tr>
<td>$R_{ac}$</td>
<td>AC conductor resistance at 25°C</td>
<td>Ω/kft</td>
</tr>
<tr>
<td>$R_{delt}$</td>
<td>temperature coefficient of AC resistance</td>
<td>Ω/°C kft</td>
</tr>
<tr>
<td>$Q_{sun}$</td>
<td>heat power input from the sun</td>
<td>kW/kft</td>
</tr>
<tr>
<td>$Q_{radiated}$</td>
<td>radiation heat losses</td>
<td>kW/kft</td>
</tr>
<tr>
<td>$Q_{convected}$</td>
<td>convection heat losses</td>
<td>kW/kft</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>processing interval or integration step</td>
<td>s</td>
</tr>
</tbody>
</table>

The equation of the line conductor temperature is expressed as a first-order differential equation:

$$\text{Power supplied to the conductor or } P - \text{Losses} = THC \frac{d TC}{dt}$$  \hspace{1cm} (1)

The heat power supplied to the conductor is expressed as

$$P = I^2 \cdot [R_{ac} + (TC - 25) \cdot R_{delt}] + Q_{sun}$$  \hspace{1cm} (2)

and the conductor heat losses can be expressed as

$$\frac{TC - TA}{TRA} = Q_{radiated} + Q_{convected}$$  \hspace{1cm} (3)

The conductor temperature differential equation is then

$$THC \frac{d TC}{dt} = I^2 \cdot [R_{ac} + (TC - 25) \cdot R_{delt}] + Q_{sun} - \frac{TC - TA}{TRA}$$  \hspace{1cm} (4)

so that the solution for the conductor temperature is provided by

$$TC(t) = \int_0^t \left[I^2 \cdot [R_{ac} + (TC - 25) - R_{delt}] + \frac{Q_{sun} - (TC - TA)}{TRA \cdot THC}ight] dt + TI$$  \hspace{1cm} (5)
This equation can be solved numerically by computing the conductor temperature increment (ΔTC) at each processing interval

\[
\Delta TC = \left[ \frac{I^2 \cdot [R_{ac} + (TC - 25) \cdot R_{del}]}{THC} + Q_{sun} - \frac{TC - TA}{TRA \cdot THC} \right] \Delta T
\]  

(6)

and by computing iteratively the new conductor temperature:

\[
TC_{new} = \Delta TC + TC_{old}
\]

(7)

At the first iteration, the conductor temperature is given the value of the initial conductor temperature:

\[
TC_1 = TI
\]

(8)

For the following iterations, the conductor temperature is calculated by computing iteratively the temperature increment using Equation (6) and then by processing the sum in Equation (7) that provides the estimated conductor temperature.

The line thermal protection can be represented as a first-order thermal model for the conductor temperature, as provided in Figure 1. In this figure, TTH is the maximum allowable temperature threshold above which the line could be tripped. TTL is an alarm threshold.

The conductor temperature time variation equation, when the conductor current (I) is a step function and Q_{sun} is assumed to be constant is provided by

\[
TC(t) = P \cdot TRA \left[ 1 - \exp \left( -\frac{t}{THC \cdot TRA} \right) \right] + TA
\]

(9)

When \( t \) becomes infinite, the conductor temperature becomes

\[
TC = \frac{I^2 \cdot [R_{ac} - 25 \cdot R_{del}]}{1 - I^2 \cdot R_{del} \cdot TRA} \cdot TRA + TA
\]

(10)

Assuming a conductor maximum temperature (TTH) the maximum current allowed on the conductor or its ampacity before the maximum temperature will be reached is

\[
I = \sqrt{\frac{TTH - TA - Q_{sun} - TRA}{[R_{ac} + (TTH - 25) \cdot R_{del}] \cdot TRA}}
\]

(11)
**COMPUTATION OF Q\textsubscript{sun}**

The heat power input ($Q_{\text{sun}}$) on the conductor is a function of the conductor longitude and latitude, the day of the year, and the local time. The methodology to compute $Q_{\text{sun}}$ is provided in [4].

Refer to Table 2 for definitions of the variables used in the computation of $Q_{\text{sun}}$.

**Table 2 Variables Used in the Computation of $Q_{\text{sun}}$**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>solar declination</td>
<td>degrees</td>
</tr>
<tr>
<td>n</td>
<td>day of the year</td>
<td>days</td>
</tr>
<tr>
<td>Hrs</td>
<td>local time in hours</td>
<td>hours</td>
</tr>
<tr>
<td>LocT</td>
<td>local time</td>
<td>degrees</td>
</tr>
<tr>
<td>Ws</td>
<td>solar time</td>
<td>degrees</td>
</tr>
<tr>
<td>SAC</td>
<td>solar absorption coefficient</td>
<td>unitless</td>
</tr>
<tr>
<td>DIA</td>
<td>conductor diameter</td>
<td>inches</td>
</tr>
<tr>
<td>z</td>
<td>solar zenith angle</td>
<td>degrees</td>
</tr>
<tr>
<td>LSTD</td>
<td>standard time meridian</td>
<td>degrees west longitude</td>
</tr>
<tr>
<td>LON</td>
<td>conductor longitude</td>
<td>degrees west longitude</td>
</tr>
<tr>
<td>LAT</td>
<td>conductor latitude</td>
<td>degrees north latitude</td>
</tr>
<tr>
<td>SIR</td>
<td>solar incident radiation</td>
<td>kW/in</td>
</tr>
<tr>
<td>$Q_{\text{sun}}$</td>
<td>solar heating</td>
<td>kW/kft</td>
</tr>
</tbody>
</table>
The heat power striking the conductor from the sun is provided by the next equation:

\[ Q_{\text{sun}} = \text{SAC} \times \text{DIA} \times \text{SIR} \]  
(12)

The solar incident radiation (SIR) is provided by way of the look-up table as a function of \( z \), the zenith angle. This look-up table is shown in Table 3.

The solar zenith angle is provided indirectly by way of its cosine as a function of the solar declination (d) the conductor latitude (LAT) and the solar time (Ws) as:

\[ \cos(z) = \sin(d) \times \sin(LAT) + \cos(d) \times \cos(LAT) \times \cos(Ws) \]  
(13)

We can, in an easier fashion, implement a polynomial approximation of a look-up table rather than putting it into the program. Based on this principle and the look-up table of Table 3, the plot of SIR as a function of the cosine of the solar zenith angle \( z \) is shown in Figure 2. A polynomial regression analysis allows fitting a polynomial of order 5 to approximate the function. The polynomial coefficients are shown in Figure 2. The solar incident radiation can then be computed using the next equation:

\[ \text{SIR} = a_5 \times \cos(z)^5 + a_4 \times \cos(z)^4 + a_3 \times \cos(z)^3 + a_2 \times \cos(z)^2 + a_1 \times \cos(z) + a_0 \]  
(14)

Power \( n \) of \( \cos(z) \) can be simply computed by multiplying \( \cos(z) \) \( n \) times by itself.

The solar time (Ws) is provided as a function of the local time (LocT) the longitude (LON) and the standard time meridian (LSTD) as

\[ Ws = \text{LocT} + (\text{LON} - \text{LSTD}) \]  
(15)

The local time (LocT) is provided through a function of the local time expressed in hours (Hrs) as

\[ \text{LocT} = (\text{Hrs} - 12) \times (-15) \]  
(16)

Finally the solar declination (d) is expressed as a function of the day of the year (n) as

\[ d = 23.45 \times \sin \left( \frac{284 + n}{365} \times 360 \right) \]  
(17)

The local time (Hrs) and the day of the year (n) are available inside the line relay as THR and DDOY. The conductor latitude (LAT) and longitude (LON) together with the standard time meridian (LSTD) have to be entered in the program as settings.

This solar model assumes no effect of clouds or shade. Hence the model calculates worst-case (maximum) heat input power from the sun. Clouds or shading will cause the estimated conductor temperature to exceed actual conductor temperature.
Table 3  SIR Value as a Function of z Look-Up Table

<table>
<thead>
<tr>
<th>z</th>
<th>SIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>1808</td>
</tr>
<tr>
<td>80</td>
<td>3350</td>
</tr>
<tr>
<td>75</td>
<td>4516</td>
</tr>
<tr>
<td>70</td>
<td>5366</td>
</tr>
<tr>
<td>65</td>
<td>5958</td>
</tr>
<tr>
<td>60</td>
<td>6416</td>
</tr>
<tr>
<td>55</td>
<td>6791</td>
</tr>
<tr>
<td>50</td>
<td>7066</td>
</tr>
<tr>
<td>45</td>
<td>7283</td>
</tr>
<tr>
<td>40</td>
<td>7500</td>
</tr>
<tr>
<td>30</td>
<td>7741</td>
</tr>
<tr>
<td>20</td>
<td>7916</td>
</tr>
<tr>
<td>10</td>
<td>7983</td>
</tr>
<tr>
<td>0</td>
<td>8033</td>
</tr>
</tbody>
</table>

Table 3: SIR Value as a Function of z Look-Up Table

\[ x = \cos(z) \]

\[ 2498x^5 - 17469x^4 + 40023x^3 - 44416x^2 + 27682x - 297 \]

Figure 2  Relation Between SIR and the Cosine of the Zenith Time (z)
**THE DETERMINATION OF THE CONDUCTOR THERMAL RESISTANCE (TRA)**

The conductor thermal resistance is provided by Equation (3) and is equal to the required temperature difference between the conductor and the ambient so that the heat power transferred from the conductor is one unit of power.

\[
\text{TRA} = \frac{\text{TC} - \text{TA}}{\text{Q}_{\text{radiated}} + \text{Q}_{\text{convected}}} \tag{18}
\]

The conductor thermal resistance to ambient (TRA) can be computed by linearizing the heat losses across the difference between two temperatures. The conductor and ambient temperatures are the first and second temperatures. The formula is as follows:

\[
\text{TRA} = \frac{\text{TC} - \text{TA}}{\text{[Q}_{\text{convected}}(\text{TC} - \text{TA}) + \text{Q}_{\text{radiated}}(\text{TC} - \text{TA})]} \tag{19}
\]

The radiated heat losses are in turn computed as

\[
\text{Q}_{\text{radiated}} = S \cdot E \cdot A \cdot (\text{KC}^4 \text{ - KA}^4) \tag{21}
\]

Where the variables are defined in Table 4.

**Table 4  Variables in Equation (21)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Stephan-Boltzman constant = 0.5275e-8</td>
<td>W/ºK·ft²</td>
</tr>
</tbody>
</table>
| E        | Thermal emissivity constant  
= 0.23 for new conductor 
= 0.91 for blackend conductor | unitless |
| A        | area of circumscribing cylinder | ft² |
| KC       | conductor temperature | ºK |
| KA       | ambient temperature | ºK |

After adjusting the units, we end up with

\[
\text{Q}_{\text{radiated}} = 0.138 \cdot \text{DIA} \cdot E \cdot \left(\frac{\text{KC}^4}{100} - \left(\frac{\text{KA}^4}{100}\right)\right) \tag{22}
\]

where DIA, the conductor diameter, is in inches and the units of \(\text{Q}_{\text{radiated}}\) are in kW/kft.

The convected heat losses can be computed according to

\[
\text{Q}_{\text{convected}} = \left[1.01 + 0.37\left(\frac{\text{DIA} \cdot R \cdot V}{H}\right)^{0.52}\right] \cdot K \cdot (\text{TC} - \text{TA}) \tag{23}
\]
where the variables are defined in Table 5.

### Table 5 Variables in Equation (23)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>air density</td>
<td>lb/ft³</td>
</tr>
<tr>
<td>V</td>
<td>air velocity</td>
<td>ft/hr</td>
</tr>
<tr>
<td>H</td>
<td>absolute viscosity</td>
<td>lb/hr ft</td>
</tr>
<tr>
<td>K</td>
<td>thermal conductivity</td>
<td>W/ft² °C</td>
</tr>
</tbody>
</table>

**THE DETERMINATION OF THE CONDUCTOR THERMAL CAPACITY (THC)**

The mass (m) of a body multiplied by its specific heat (C<sub>s</sub>) is known as C, the thermal capacity of the system with units in kJ/°C. It represents the amount of energy in joules required to raise the system temperature by one degree Celsius.

In the case of a power conductor, the thermal heat capacity is simply determined by multiplying the number of pounds of aluminum (WA) by the aluminum-specific heat and adding the number of pounds of steel (WS) multiplied by the steel-specific heat:

\[
THC = \frac{WA \cdot 428.8 + WS \cdot 204.9}{1000}
\]

(24)

**SETTINGS**

There are 18 settings that have to be entered into the line thermal relay algorithm. They must all be scaled to a per unit conductor length equal to 1000 ft. These settings can be broken into four groups: Solar Model Settings, Thermal Model Settings, Temperature Settings, and Logic Switches.

### Solar Model Settings

### Table 6 Solar Model Settings

<table>
<thead>
<tr>
<th>Setting</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSH</td>
<td>default solar heating</td>
<td>kW/kft</td>
</tr>
<tr>
<td>SAC</td>
<td>solar absorption coefficient</td>
<td>unitless</td>
</tr>
<tr>
<td>DIA</td>
<td>conductor diameter</td>
<td>inches</td>
</tr>
<tr>
<td>LSTD</td>
<td>longitude of time standard</td>
<td>degrees west longitude</td>
</tr>
<tr>
<td>LON</td>
<td>longitude of conductor</td>
<td>degrees west longitude</td>
</tr>
<tr>
<td>LAT</td>
<td>latitude of conductor</td>
<td>degrees north latitude</td>
</tr>
</tbody>
</table>
The default solar heating (DSH) has to be set to some constant value if the sun radiated power \( Q_{\text{sun}} \) is not computed according to Equation (12) but introduced as a constant value equal to DSH.

The only purpose of entering the diameter (DIA) as a setting is to compute \( Q_{\text{sun}} \) as provided by Equation (12). In reality, a corrected diameter \( \text{DIA}_{\text{cor}} \) has to be entered in order to take into account the elevation and the atmosphere quality as in

\[
\text{DIA}_{\text{cor}} = k_1 \cdot k_2 \cdot \text{DIA}
\]  

(25)

\( k_1 \) and \( k_2 \) must be chosen according to Table 7 and Table 8.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Values of ( k_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elevation (ft)</strong></td>
<td><strong>K(_1)</strong></td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>5000</td>
<td>1.15</td>
</tr>
<tr>
<td>10000</td>
<td>1.25</td>
</tr>
<tr>
<td>15000</td>
<td>1.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8</th>
<th>Values of ( k_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Atmosphere</strong></td>
<td><strong>K(_2)</strong></td>
</tr>
<tr>
<td>Clean</td>
<td>1.00</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.82</td>
</tr>
</tbody>
</table>

The longitude of the time standard must be entered according to Table 9.

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Longitude of the Time Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Zone</strong></td>
<td><strong>Meridian</strong></td>
</tr>
<tr>
<td>Eastern</td>
<td>75°W</td>
</tr>
<tr>
<td>Central</td>
<td>90°W</td>
</tr>
<tr>
<td>Mountain</td>
<td>105°W</td>
</tr>
<tr>
<td>Pacific</td>
<td>120°W</td>
</tr>
</tbody>
</table>
**Thermal Model Settings**

**Table 10  Thermal Model Settings**

<table>
<thead>
<tr>
<th>Setting</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ac}$</td>
<td>AC resistance at $25^\circ C$</td>
<td>$\Omega$/kft</td>
</tr>
<tr>
<td>$R_{delt}$</td>
<td>temperature coefficient of AC resistance</td>
<td>$\Omega$/°C kft</td>
</tr>
<tr>
<td>THC</td>
<td>thermal heat capacity</td>
<td>kJ/°C kft</td>
</tr>
<tr>
<td>TRA</td>
<td>thermal resistance to ambient</td>
<td>°C kft/kW</td>
</tr>
</tbody>
</table>

These four settings must be calculated from the conductor characteristic tables or the provided formulas. See the example below.

**Temperature Settings**

**Table 11  Temperature Settings**

<table>
<thead>
<tr>
<th>Setting</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>EAT</td>
<td>estimated ambient temperature</td>
<td>°C</td>
</tr>
<tr>
<td>EOT</td>
<td>estimated offset temperature</td>
<td>°C</td>
</tr>
<tr>
<td>TTH</td>
<td>high temperature threshold</td>
<td>°C</td>
</tr>
<tr>
<td>TTL</td>
<td>low temperature threshold</td>
<td>°C</td>
</tr>
<tr>
<td>TI</td>
<td>conductor initial temperature</td>
<td>°C</td>
</tr>
</tbody>
</table>

If an external temperature sensor is not available, the estimated ambient temperature (EAT) must be entered. This will introduce errors into the thermal model because the ambient temperature is not measured but assumed to be equal to some value.

The estimated offset temperature (EOT) is the temperature difference between the ambient temperature available (introduced or measured) and the estimated hottest ambient temperature along the line. It must be entered as a number different from zero if it is estimated that the difference indeed exists. If not, it must be equal to zero.

The two thresholds (TTH and TTL) are used either to trip the line or to generate an alarm, respectively.

The initial temperature setting (TI) depends upon the line state at the moment the function will be started. If the line is open, TI can be given simply, by neglecting the impact of the sun, the value of the ambient temperature. If a current is flowing into the line, Equation (10) must be used to compute the conductor temperature, and this temperature becomes the initial temperature setting.

Note that the accuracy of the initial temperature setting is not critical because the conductor temperature will adjust itself within the time frame of the thermal model time constant.
**Logic Switches**

**Table 12  Local Switch Settings**

<table>
<thead>
<tr>
<th>Setting</th>
<th>Definition</th>
<th>Logic States</th>
</tr>
</thead>
<tbody>
<tr>
<td>THE</td>
<td>thermal trip enable</td>
<td>1/0</td>
</tr>
<tr>
<td>TSE</td>
<td>thermal sensor enable</td>
<td>1/0</td>
</tr>
<tr>
<td>SGE</td>
<td>solar generator enable</td>
<td>1/0</td>
</tr>
</tbody>
</table>

THE must be set to 1 if the line ought to be tripped due to excessive temperature.

TSE must be set to 1 if an external sensor is available for the ambient temperature. Otherwise, the ambient temperature is assumed to be equal to EAT.

SGE must be set to 1 if \( Q_{\text{sun}} \) is computed from Equation (12). It is set to 0 if \( Q_{\text{sun}} \) is introduced as being equal to a constant equal to DSH.

**EXAMPLE OF IMPLEMENTATION**

A line conductor is made of Drake cable type with the following basic characteristics that can be extracted from tables:

- Diameter: \( \text{DIA} = 1.108 \text{ in} \) (\( k_1=k_2=1.0 \))
- Thermal emissivity constant: \( E = 0.5 \) (unitless)
- AC resistance per mile at 25°C: \( R_L = 0.117 \Omega \text{/mile} \)
- AC resistance per mile at 75°C: \( R_H = 0.139 \Omega \text{ /mile} \)
- Weight of aluminum 1000 ft of conductor: \( W_A = 750 \text{ lb/kft} \)
- Weight of steel of 1000 ft of conductor: \( W_S = 344 \text{ lb/kft} \)

**Determination of \( R_{ac} \)**

The value of \( R_{ac} \), the conductor resistance per 1000 ft, is determined from the value of \( R_L \):

\[
R_{ac} = \frac{R_L}{5.28} = \frac{0.117}{5.28} = 0.022159 \text{ } \Omega / \text{kft}
\]  

**Determination of \( R_{\text{delt}} \)**

The value of \( R_{\text{delt}} \), the conductor temperature coefficient, is computed from

\[
R_{\text{delt}} = \frac{R_H - R_L}{(75 - 25) \cdot 5.28} = \frac{0.139 - 0.117}{50 \cdot 5.28} = 0.0000833 \text{ } \Omega / \text{°C kft}
\]
**Determination of THC**

The thermal heat capacity is determined by multiplying the number of pounds of aluminum (WA) by the aluminum-specific heat and adding the number of pounds of steel (WS) multiplied by the steel-specific heat:

\[
THC = \frac{WA \cdot 428.8 + WS \cdot 204.9}{1000} = 392.086 \text{ kJ/}^\circ\text{C kft}
\]  

(28)

**Determination of TRA**

TRA is computed using Equation (18) through Equation (23) and using a conductor temperature of 90°C and an ambient temperature of 40°C.

First, we compute the radiated power. From Equation (22), we have

\[
Q_{\text{radiated}} = 0.138 \cdot E \cdot DIA \left( \frac{KC}{100} \right)^4 - \left( \frac{KA}{100} \right)^4
\]

or

\[
Q_{\text{radiated}} = 0.138 \cdot 0.5 \cdot 1.108 \left[ \frac{363}{100} \right]^4 - \left[ \frac{313}{100} \right]^4 = 5.937 \text{ kW/kft}
\]

(30)

The convected power is computed from Equation (23) with the assumed air constants listed in Table 13.

**Table 13 Assumed Air Constants**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>air density</td>
<td>0.0752 lb/ft³ (20°C, sea level)</td>
</tr>
<tr>
<td>V</td>
<td>air velocity</td>
<td>2 ft/sec = 7200 ft/hr</td>
</tr>
<tr>
<td>H</td>
<td>absolute viscosity</td>
<td>0.0439 lb/hr ft (20°C)</td>
</tr>
<tr>
<td>K</td>
<td>thermal conductivity</td>
<td>0.00784 W/ft² °C (20°C)</td>
</tr>
</tbody>
</table>

\[
Q_{\text{conv}} = \left[ 1.01 + 0.371 \left( \frac{1.108 \cdot 0.0752 \cdot 7200}{0.0439} \right)^{0.52} \right] \cdot 0.00784 \cdot (90 - 40)
\]

\[
= 20.964 \text{ kW/kft}
\]

(31)

Finally, we have for TRA:

\[
TRA = \frac{TC - TA}{Q_{\text{radiated}} + Q_{\text{convected}}} = \frac{90 - 40}{5.937 + 20.964} = 1.859^\circ\text{C kft/kW}
\]

(32)

The time constant of the first-order thermal model is provided then as

\[
\text{Thermal Time Constant} = THC \cdot TRA = 392.086 \cdot 1.859 = 728.9 \text{ s} = 12.15 \text{ mn}
\]

(33)
The practical meaning of this time constant is that if a step current is applied to the conductor, it will take 12.15 minutes for the conductor to reach 63 percent of the new steady-state temperature value.

**Final Settings**

Let us assume the line is located in the Montreal (Canada) area. Assume the routine is started with the line open in summer time. We get the final set of settings that has to be introduced into the routine as constants.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSH</td>
<td>default solar heating</td>
<td>0</td>
</tr>
<tr>
<td>SAC</td>
<td>solar absorption coefficient</td>
<td>0.5</td>
</tr>
<tr>
<td>DIA</td>
<td>conductor diameter</td>
<td>1.108</td>
</tr>
<tr>
<td>LSTD</td>
<td>longitude of time standard</td>
<td>75.00</td>
</tr>
<tr>
<td>LON</td>
<td>longitude of conductor</td>
<td>73.43</td>
</tr>
<tr>
<td>LAT</td>
<td>latitude of conductor</td>
<td>45.56</td>
</tr>
<tr>
<td>Rac</td>
<td>AC resistance at 25°C</td>
<td>0.022159</td>
</tr>
<tr>
<td>Rdelt</td>
<td>temperature coefficient of AC resistance</td>
<td>0.0000833</td>
</tr>
<tr>
<td>THC</td>
<td>thermal heat capacity</td>
<td>392.086</td>
</tr>
<tr>
<td>TRA</td>
<td>thermal resistance to ambient</td>
<td>1.859</td>
</tr>
<tr>
<td>EAT</td>
<td>estimated ambient temperature</td>
<td>0</td>
</tr>
<tr>
<td>EOT</td>
<td>estimated offset temperature</td>
<td>0</td>
</tr>
<tr>
<td>TH</td>
<td>high temperature threshold</td>
<td>90</td>
</tr>
<tr>
<td>TL</td>
<td>low temperature threshold</td>
<td>80</td>
</tr>
<tr>
<td>TI</td>
<td>conductor initial temperature</td>
<td>25</td>
</tr>
<tr>
<td>THE</td>
<td>thermal trip enable</td>
<td>1</td>
</tr>
<tr>
<td>TSE</td>
<td>thermal sensor enable</td>
<td>1</td>
</tr>
<tr>
<td>SGE</td>
<td>solar generator enable</td>
<td>1</td>
</tr>
</tbody>
</table>

The thermal model implemented for this example is shown in Figure 3. PCT10 and PCT11 are two conditioning timers with both a pickup and a dropout time of 10 cycles.
TRANSMISSION LINE RELAY PROGRAMMING CAPABILITIES

The transmission line relay on which the line thermal protection element is implemented has user-friendly programming capabilities consisting of mathematical and logical equations using a language similar to BASIC. It is beyond the scope of this paper to describe the complete range of programming capabilities, but the following features have been used for the purpose of implementing the line thermal algorithm:

1. Sixty-four floating-point mathematical variables noted PMV01–PMV64 are available for the purpose of equation evaluation or memory storage. The four basic algebraic operations (addition, subtraction, multiplication, and division) are applicable on these variables together with the algebraic functions abs (absolute value of), sin, cos, sqrt, log, acos, asin, and ln.

2. Sixty-four Boolean variables represented as PSV01–PSV64 are available. All basic Boolean operation as OR, AND, NOT are available. Comparisons (> , > , = , <= , >= , < > ) are applicable to floating-point variables and their outputs are defined as Boolean variables.

3. Sixteen conditioning timers noted PCT01–PCT16 are used to condition Boolean values. PCT01IN represents the logic input, PCT01PU the pickup time in cycles, PCT01DO the dropout time in cycles, and PCT01Q the logical output.

4. Analog values of phase currents represented as LIARMS, LIBRMS, and LICRMS are available as 10-cycle averaged root-mean-squared (rms) quantities.

5. Analog real-time variables such as day-of-the-year, hour, and minutes are available and represented as DDOY, THR, and TMIN, respectively.

6. Analog values for temperature measurements are acquired by an external module and noted RTD01–RTD12. Twelve RTDs can be attached to the module that transmits to the line relay at the rate of two sets of measurements per second by way of an optical fiber (see Figure 4).
Temperature measurements are represented as RTD01–RTD12. For the purpose of this application, RTD01 is the ambient temperature measurement.

![Figure 4 Ambient Temperature Measurement Principle](image)

**PROGRAM FLOW CHART**

The simplified program flow chart is shown in Figure 5.

1. The first time the routine is processed (detected by PFRTEX = 1), the initial conductor temperature is entered.

2. If the setting SGE = 1, the heat power from the sun is computed; otherwise, it is set to the value DSH.

3. The maximum of the three-phase currents is then chosen as the conductor current.

4. The temperature increment is then computed by implementing Equation (6). The conductor temperature is then integrated with an integration step equal to 1/8 of a cycle or 0.0020833 s at a nominal frequency of 60 Hz. This integration step corresponds to the routine internal processing rate.

5. The routine then simply verifies if the computed temperature is above the alarm threshold or the tripping threshold. Before tripping or triggering an alarm, the signals are processed through two conditioning timers with a pickup and a dropout time of 10 cycles.
First process?

Yes → TC = TI

No → SGE = 1?

Yes → Compute $Q_{\text{sun}}$ (Eq. 12)

No → $Q_{\text{sun}} = \text{DSH}$

Assess highest phase current IL

Compute temperature increment DTC (Eq. 6)

$TC(\text{new}) = TC(\text{old}) + DTC$

$TC > THL$

Yes → Trip line if $THE = 1$

No → $TC > TLL$

Yes → Set $\text{ALARM} = 1$

No → End

Figure 5  Simplified Line Thermal Model Flow Chart
PROGRAM IMPLEMENTATION

An example of the routine coding as it would be implemented by a user is shown in Figure 6 and Figure 7. Note that power measurements are made in kW and that temperature measurements are in degrees Celsius.

1. Lines 1–19 introduce all the line thermal element settings.

2. Lines 20–39 implement the computation of $Q_{\text{sun}}$ or the heat power from the sun following Equation (12) through Equation (17). Note that $Q_{\text{sun}}$ has to be positive (daytime) or zero (nighttime).

3. Lines 40–42 introduce the conductor initial temperature value.

4. Lines 43–46 perform the maximum phase current determination.

5. Lines 47–48 establish the ambient temperature depending on if we use a set value or a temperature measurement.

6. Lines 49–52 establish the conductor temperature by implementing Equation (6) and Equation (7).

7. Lines 53–55 determine the alarm output.

8. Lines 55–58 determine the trip output.

1     PMV03 := 70.430000                               #CONDUCTOR LONGITUDE
2     PMV04 := 75.000000                               #LSTD
3     PMV05 := 0.500000                                #SAC
4     PMV06 := 0                                       #VALUE OF DEFAULT SOLAR HEATING
5     PMV07 := 0                                       #ESTIMATED OFFSET TEMPERATURE
6     PMV08 := 1.000000                                #DIAMETER
7     PMV09 := 45.560000                               #CONDUCTOR LATITUDE
8     PMV10 := 25                                     #VALUE OF INITIAL TEMPERATURE
9     PMV11 := 0.02216                                 #VALUE OF RAC CONDUCTOR RESISTANCE
10    PMV12 := 0.00008333                              #VALUE OF RDELT TEMPERATURE COEFFICIENT
11    PMV13 := 392.086                                 #VALUE OF THC THERMAL HEAT CAPACITY
12    PMV14 := 1.859                                   #VALUE OF TRA THERMAL RESISTANCE TO AMBIENT
13    PMV15 := 90                                     #VALUE OF TH HIGH TEMPERATURE THRESHOLD
14    PMV16 := 80                                     #VALUE OF TL LOW TEMPERATURE THRESHOLD
15    PMV17 := 0                                       #ESTIMATED AMBIENT TEMPERATURE
16    PMV18 := RTD01                                   #VALUE OF A TA AMBIENT TEMPERATURE
17    PSV05 := 1                                       #STATE OF SGE SOLAR GENERATOR ENABLE
18    PSV06 := 1                                       #STATE OF THERMAL SENSOR ENABLE
19    PSV18 := 1                                       #STATE OF THERMAL TRIP ENABLE
20    PMV01 := DDOY                                   #DAY OF THE YEAR
21    PMV02 := THR + TMIN * 0.0166667                  #HOURS OF THE DAY
22    PMV20 := 23.450001 * SIN((284.000000 + PMV01) * 0.98630137)                    #SUN DECLINATION
23    PMV21 := (PMV02 - 12.000000) * (-15.000000)                                    #LOCT
24    PMV22 := PMV21 + (PMV03 - PMV04)                  #WS
25    PMV24 := SIN(PMV20) * SIN(PMV09) + COS(PMV20) * COS(PMV09) * COS(PMV22)        #COS(Z)
26    PMV25 := PMV24 * PMV24
27    PMV27 := 27682.000000 + COS(PMV20) + COS(PMV09) * COS(PMV22) * PMV25 + 2498 * PMV25
28    PMV25 := PMV24 + 2498 * PMV25
29    PMV27 := PMV27 - 17469 * PMV25
30    PMV25 := PMV25 * PMV25
31    PMV27 := PMV27 + 40023 * PMV25
32    PMV25 := PMV25 * PMV24
33    PMV27 := PMV27 + 2498 * PMV25

(continued on next page)
34 PMV28 := PMV27 #SIR VALUE
35 PMV29 := PMV05 * PMV08 * PMV28 #QSUN VALUE
36 PSV01 := PMV29 >= 0
37 PMV30 := (PMV29 * 0.001) * PSV01
38 PSV08 := NOT PSV05
39 PMV30 := ((PMV29 * 0.001) * PSV01) * PSV05 + PMV06 * PSV08 #QSUN VALUE

Figure 6  Routine Coding: Introduction of Settings and Computation of Qsun

40 PSV02 := PFRTEX #DETECTION OF FIRST PROCESSING INTERVAL
41 PSV03 := NOT PFRTEX
42 PMV35 := PMV10 * PSV02 + PMV35 * PSV03 #INTRODUCTION OF THE TC INITIAL VALUE
43 PSV10 := (LIARMS >= LIBRMS) AND (LIARMS >= LICRMS) #STATE OF PHASE A LARGEST CURRENT
44 PSV11 := ((LIBRMS >= LIARMS) AND (LIBRMS > LICRMS)) OR ((LICRMS >= LIARMS) AND (LIBRMS >= LICRMS))
45 PSV12 := ((LICRMS >= LIARMS) AND (LICRMS > LIBRMS)) OR ((LICRMS > LIARMS) AND (LICRMS >= LIBRMS))
46 PMV19 := LIARMS * PSV10 + LIBRMS * PSV11 + LICRMS * PSV12 #CHOICE OF GREATEST RMS PHASE CURRENT
47 PSV07 := NOT PSV06
48 PMV32 := (PMV18 * PSV06 + PMV17 * PSV07) + PMV07 #VALUE OF AMBIENT TEMPERATURE
49 PMV36 := ((PMV19 * PMV19) * (PMV11 + (PMV35 - 25) * PMV12)) * 0.001
50 PMV37 := ((PMV36 + PMV30) / PMV13) - ((PMV35 - PMV32) / (PMV13 * PMV14))
51 PMV38 := PMV37 * (0.00208333) #TEMPERATURE INCREMENT
52 PMV35 := PMV35 + PMV38 #TEMPERATURE INTEGRATION
53 PCT10IN := PMV35 > PMV16 #DETECTION OF ALARM STATE
54 PCT10PU := 10
55 PCT10DO := 10
56 PCT11IN := (PMV35 > PMV15) AND PSV18 #DETECTION OF TRIP STATE
57 PCT11PU := 10
58 PCT11DO := 10

Figure 7  Routine Coding: Computation of Conductor Temperature and Element Outputs

Figure 8 shows the measured conductor temperature rise when subjected to a primary current step of 791 A over an interval of time of 30 mn. The temperature decay occurs when the current step is removed. For this experiment, Qsun has been set to zero, TA has been set to a fixed value of 23°C, and the thermal time constant is set to 5 mn (TRA = 1.859 and THC = 161.38). The maximum temperature reached is 51.09°C when the theoretical value as provided by Equation (10) is 51.33°C.
CONCLUSIONS

In this paper, it has been shown that a reliable line thermal protection element can be implemented by assuming the conditions of a worst-case scenario: the line temperature is computed on the line section assumed to have the greatest external ambient temperature and exposed to the maximum solar heating.

The element can be implemented into a transmission line relay with programmable mathematical and logical equation capabilities. Necessary analog quantities are phase currents, real-time variables like day-of-the-year, hours, and minutes, and finally an external measurement of the ambient temperature.

REFERENCES


BIOGRAPHIES

**Gabriel Benmouyal, P.E.,** received his B.A.Sc. in Electrical Engineering and his M.A.Sc. in Control Engineering from Ecole Polytechnique, Université de Montréal, Canada in 1968 and 1970, respectively. In 1969, he joined Hydro-Québec as an Instrumentation and Control Specialist. He worked on different projects in the field of substation control systems and dispatching centers. In 1978, he joined IREQ, where his main field of activity was the application of microprocessors and digital techniques to substation and generating-station control and protection systems. In 1997, he joined Schweitzer Engineering Laboratories in the position of Research Engineer. He is a registered professional engineer in the Province of Québec, is an IEEE Senior Member, and has served on the Power System Relaying Committee since May 1989. He is the author or co-author of several papers in the field of signal processing and power networks protection.

**Michael B. Bryson** received his BSEE from the University of Idaho in 1989. He joined Schweitzer Engineering Laboratories as a Development Engineer in 1989. He has held the position of Power Engineer since 2001. His interests include development and testing of digital protection algorithms, signal processing, control systems, and real-time digital simulation of power systems.

**Marc Palmer** received his BE in electrical engineering from the University of Canterbury, Christchurch, New Zealand in 1994. He has worked for the Electricity Corporation of New Zealand (1994–1998), Mighty River Power Limited (1999), and Meridian Energy (1999–2001), all of which are generation utilities in New Zealand. At Meridian Energy, he was the Senior Control and Instrumentation Consultant. In May 2001, he joined Schweitzer Engineering Laboratories in the position of Field Application Engineer.