

# Field Evaluation of Slip-Dependent Thermal Model for Motors With High-Inertia Starting

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# FIELD EVALUATION OF SLIP-DEPENDENT THERMAL MODEL FOR MOTORS WITH HIGH-INERTIA STARTING

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**Abstract**—This paper evaluates the advantages of using the slip-based thermal model with data collected from an actual field installation. The paper presents information on motor starts before and after installation of the new motor protection technology. Improvement claims are based on the first order thermal model, enhanced with the calculation of slip and rotor resistance to improve accuracy of its starting element. This improved accuracy calculates more precise per-unit rotor temperature, allowing longer start times than other methods. The new model uses current, voltage, and algorithm to calculate slip and rotor resistance from motor apparent impedance. The evaluation includes a start report from a field installation.

## I. INTRODUCTION

One troublesome area in motor protection is protecting the start of an ac motor coupled to a high-inertia mechanical load without prematurely tripping it offline.

Traditionally, solving this difficulty involves the use of a motor-mounted speed switch in the starting protection scheme.

This approach generates mixed responses from owners of high-inertia load applications from complete acceptance, to reserved acceptance, to complete rejection of speed-switch usage.

The most frequent reasons given by those who oppose the use of speed switches in start protection logic are the following:

- Once the speed-switch contact changes states at a pre-set speed, there is no way of knowing that the motor continues accelerating towards its rated speed.
- Reliability of speed switches.
- Additional cost of speed-switch assembly and maintenance.

This paper presents field results from high-inertia starting motor installations where the motor protection relay has a unique slip-dependent thermal model.

The slip-dependent thermal model foregoes the need for a speed switch, yet offers start protection without false tripping during the starting process.

To maintain a practical flavor, this paper avoids detailed equation development. Thorough development of the equations is discussed in the paper “Optimizing Motor Thermal Models,” [1] authored by the designer of the slip-dependent thermal model, Mr. Stanley E. Zocholl.

## II. OVERVIEW OF THE FIRST ORDER THERMAL MODEL

Fig. 1 illustrates the first order thermal model. The major components of the model are listed below:

- Heat Source. Heat flow from the source is  $I^2r$  watts (J/s).
- Thermal Capacitance ( $C_{th}$ ) represents a motor with a thermal capacity  $C_{th}$  to absorb heat from the heat source. The unit of thermal capacitance is  $J/^\circ C$ .

- Thermal Resistance ( $R_{th}$ ) represents the heat dissipated by a motor to its surroundings. The unit of thermal resistance is  $^\circ C/W$ .
- Comparator compares the calculated motor pu temperature with a preset value based on the motor manufacturer’s data.

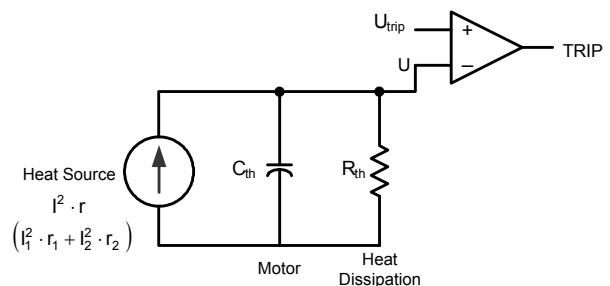


Fig. 1 First Order Thermal Model

The qualitative analysis of this model states that heat produced by the heat source is transferred to the motor, which in turn, dissipates the heat to the surrounding environment.

The quantitative analysis is defined by a first order linear differential equation similar to a parallel RC electrical circuit and is:

$$I^2r = C_{th} \cdot \frac{dU}{dt} + \frac{U}{R_{th}} \quad (W) \quad (1)$$

Where:

$$I^2r = \text{Total heat flow (W)}$$

$$C_{th} \cdot \frac{dU}{dt} = \text{Heat flow into motor (W)}$$

$$\frac{U}{R_{th}} = \text{Heat flow into surroundings (W)}$$

The solution to this equation is of no relevance to the topic at hand; however, it can be found in [1].

A further step was taken in refining the above thermal model. Motors are comprised of two major components—stator and rotor.

The stator produces a rotating magnetic field (at line frequency) in the air gap and induces voltage in the rotor bars. This induced voltage, in turn, produces current flow in the rotor bars. The rotor current produces a magnetic field of its own.

The rotor magnetic field is at  $90^\circ$  to the air-gap magnetic field, thus generating torque tangential to the rotor surface, producing rotational force, which turns the shaft.

Because the construction of the stator and rotor is different, so is their thermal characteristic.

To accommodate this major difference in stator and rotor thermal properties, the first order thermal model was refined into the following two elements:

- The starting element protects the rotor during the starting sequence.
- The running element protects the stator when the motor is up to speed and running.

Transition from one element to the other was set at 2.5 times the rated full load current of the motor. The high-inertia starting solution, using the slip-dependent thermal model, affects the rotor element design only. The stator element is not discussed in this paper.

### III. APPLYING FIRST ORDER THERMAL MODEL TO MOTOR STARTING

Starting an ac motor is regarded as an adiabatic (lossless) process.

Starting deposits an immense amount of heat (up to one hundred times the rated heating) in rotor bars. Also, the duration of the starting sequence is magnitudes shorter than motor thermal time constants. Thus it is regarded that any heat deposited in the rotor will not dissipate to the surroundings during the starting sequence (it will dissipate later when the motor is up to speed and running).

If we apply this assumption to the first order thermal model depicted in Fig. 1, we are effectively saying that the thermal resistance of the motor during starting is infinity ( $R = \infty$ ).

Substituting this condition into (1) yields:

$$I^2 \cdot r = C_{th} \cdot \frac{dU}{dt} + \frac{U}{\infty} = C_{th} \cdot \frac{dU}{dt} \quad (W) \quad (2)$$

Rearranging (2):

$$dU = \frac{I^2 \cdot r}{C_{th}} \cdot dt \quad (^\circ C)$$

The thermal capacity of the motor is a physical attribute and does not change. Motor resistance is assumed to be constant at this point. Convert the above expression to pu quantity by substituting the following:

$$r = C_{th} = 1 \text{ pu}$$

Thus:

$$dU = I^2 \cdot dt$$

Integrating the expression:

$$\int dU = \int I^2 \cdot dt = I^2 \int dt$$

The solution to this general integral is:

$$U = I^2 \cdot t \quad (\text{pu } ^\circ C) \quad (3)$$

Motor manufacturers supply thermal limit information as part of motor data.

The starting thermal limit is expressed in terms of the maximum time (motor safe stall time) that corresponding locked rotor current can be applied to a motor.

Applying this to (3):

$$I = I_{LRA} \text{ (pu locked rotor amperes)}$$

$$t = T_{STALL} \text{ (safe stall time, seconds)}$$

$$U = I_{LRA}^2 \cdot T_{STALL} \quad (\text{pu } ^\circ C) \quad (4)$$

Incorporating all of these changes to Fig. 1 results in the  $I^2t$  element of the first order thermal model as illustrated in Fig. 2.

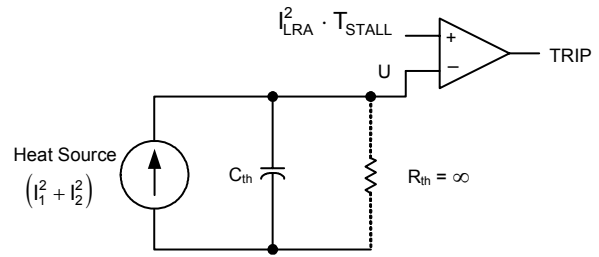


Fig. 2  $I^2t$  Starting Element

If we were to plot the pu temperature response of this model versus the line current of the motor, the response curve would be a straight line, as illustrated in Fig. 3.

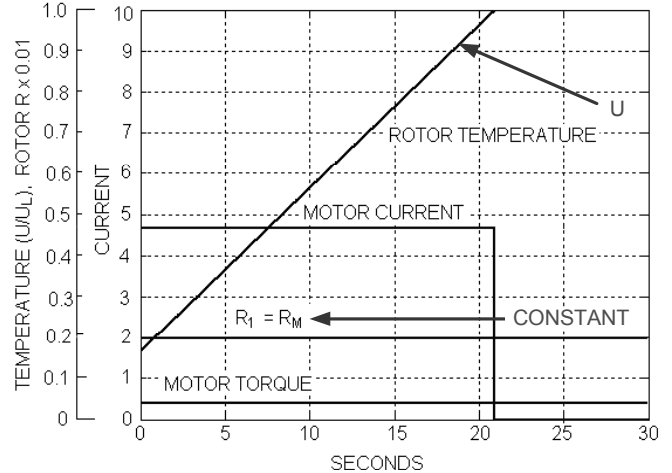


Fig. 3  $I^2t$  Starting Element Response Curve

Please note that this model keeps the rotor resistance constant at  $R_M$ , which occurs at stand still ( $S = 1.0$ ) and is considered 1 pu.

### IV. HIGH-INERTIA STARTING

The  $I^2t$  starting element performs well except for high-inertia starts.

High-inertia start is a start where the time to accelerate a motor up to rated speed is equal to or longer than its specified safe stall time.

When trying to use the above starting element in high-inertia cases, the starting thermal limit of  $I_{LRA}^2 \cdot T_{STALL}$  is reached before the current drops below  $2.5 \cdot FLA$ , resulting in premature tripping of the motor as shown in Fig. 3. This would occur during every start. In other words, the motor cannot be started successfully. Solution? Speed switch.

As noted in Section I, the reluctance of some customers to use speed switches led to the development of a starting element that dispenses using a speed switch in a motor starting logic scheme, while providing complete protection during this critical period.

## V. INDUCTION MACHINE EQUIVALENT CIRCUIT

At the dawn of the twentieth century, an engineer and inventor named Charles Steinmetz came up with an equivalent circuit for induction machines. It is still the only equivalent circuit used today and is illustrated in Fig. 4.

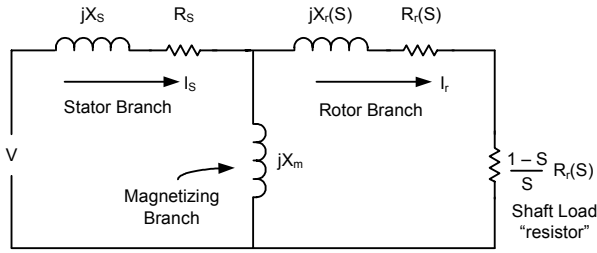


Fig. 4 Induction Machine Equivalent Circuit

A closer look at the circuit reveals several interesting observations that might not be obvious at first glance:

- The stator and magnetizing branches are always connected across the supply voltage, thus exposed to line frequency at all times. The numerical value of stator impedance is fixed.
- The rotor branch is not connected to the line voltage. Instead, it is exposed to the voltage induced by the air gap magnetic field. This voltage frequency depends on how fast rotor bars cut the magnetic field. The numerical value of rotor impedance varies with the induced voltage frequency. This frequency is the difference between the electrical frequency of the line voltage and the mechanical rotational speed of the rotor/shaft assembly. The difference is called slip and is denoted  $S$ .
- Rotor impedance is slip-dependent.
- A fact that is not evident from examining the equivalent circuit is that magnetic flux (and thus current) distribution in the rotor bar varies with rotor speed.
- At stand still, only approximately one-third of the rotor bar's cross section conducts rotor current.
- At rated speed, most of the rotor bar's cross section is used.
- This fact influences rotor resistance,  $R_r$ .
- Rotor resistance at standstill is approximately three times larger than at rated speed.
- $R_r$  is speed-dependent and is denoted  $R_r(S)$  in Fig. 4.
- This particular phenomenon is the key ingredient that makes the slip-dependent thermal model possible.

## VI. CALCULATING MOTOR SLIP

Let us calculate the input impedance of the induction machine equivalent circuit. After calculating and separating it into its real and imaginary components, the real part represents the input resistance of an induction machine.

Motor input resistance  $R$  is:

$$R = R_s + \frac{R_r(S)}{A \cdot S} \quad (\text{pu } \Omega) \quad (5)$$

Where:

- $R$  = Measured motor input resistance
- $R_s$  = Stator resistance
- $R_r(S)$  = Rotor resistance
- $S$  = Motor slip
- $A = 1.2$  (empirical constant)

Reference [1] shows the detailed derivation of  $R$ . It also derives the expression for  $R_r(S)$  in terms of maximum rotor resistance  $R_M$ , which occurs at standstill ( $S = 1$ ) and normal rotor resistance  $R_N$ , which occurs at rated motor speed ( $S = \text{rated}$ ).

The expression is:

$$R_r(S) = (R_M - R_N) \cdot S + R_N \quad (\text{pu } \Omega) \quad (6)$$

Substituting (6) into (5) and solving for  $S$  yields:

$$S = \frac{R_N}{A \cdot (R - R_s) - (R_M - R_N)} \quad (7)$$

Two quantities are required by the slip-dependent thermal model in order to calculate fixed parameters  $R_s$ ,  $R_M$ , and  $R_N$ :

FLS = Full load slip

LRQ = Locked rotor torque (at  $S = 1$ )

When FLS and LRQ are entered as set points, the algorithm automatically calculates  $R_M$  and  $R_N$  and stores them in nonvolatile memory.

The moment the motor is energized, the relay takes the first snapshot of motor input impedance  $Z$ , extracts the real component  $R$ , and calculates  $R_s$ .

The equations for  $R_M$ ,  $R_N$ , and  $R_s$  are:

$$R_M = \frac{\text{LRQ}}{\text{LRA}^2} \quad (\text{pu } \Omega) \quad (8)$$

$$R_N = \frac{Q_{\text{RATED}} \cdot S}{I_{\text{FLA}}^2} = \frac{1 \cdot \text{FLS}}{1^2} = \text{FLS} \quad (\text{pu } \Omega) \quad (9)$$

$$R_s = R - \frac{R_M}{1.2 \cdot 1} = R - 0.833 \cdot R_M \quad (\text{pu } \Omega) \quad (10)$$

Now slip is solely a function of motor input resistance  $R$ .

$R$  is measured and updated every 4 ms.

$S$  is calculated every two line frequency cycles.

A study using 2,250-hp induction motor data illustrates the relationship between motor input resistance  $R$ , motor slip  $S$ , and rotor resistance  $R_r(S)$ .

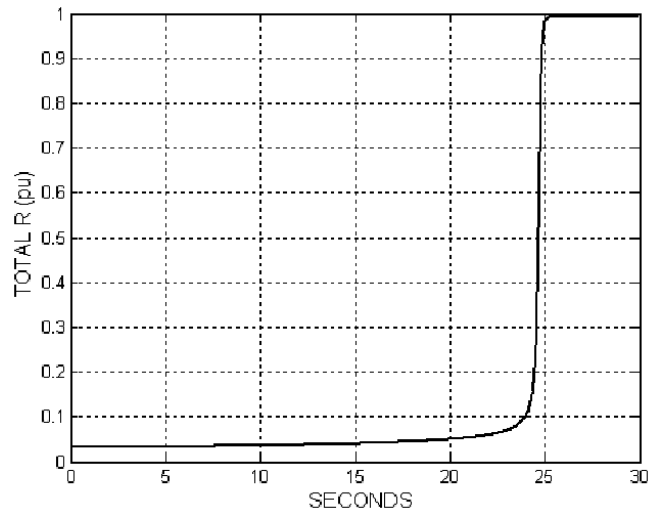


Fig. 5 Motor Input Resistance During Starting

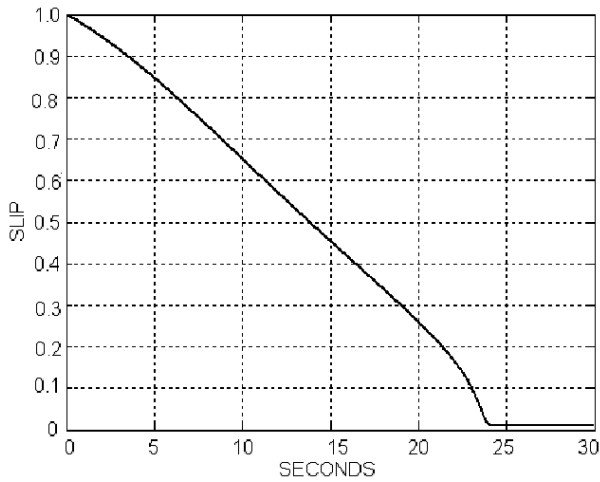


Fig. 6 Motor Slip During Starting

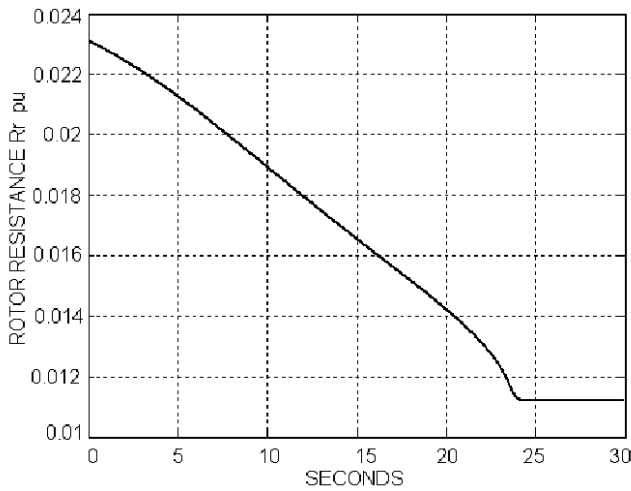


Fig. 7 Rotor Resistance During Starting

Figs. 5, 6, and 7 clearly illustrate the close dependence of rotor resistance  $R_r(S)$  on motor slip  $S$  that, in turn, is extracted from motor input resistance  $R$ .

### VII. MODIFYING THE STARTING ELEMENT

The final step in establishing the slip-dependent thermal model is incorporating the slip-dependent rotor resistance  $R_r(S)$  into the heat source of the thermal model shown in Fig. 2.

As mentioned before, the standard starting thermal element assumes a constant rotor resistance of 1 pu across the entire speed range from stand still ( $S = 1$ ) to rated speed at rated shaft output ( $S = \text{FLS}$ ). That value is  $R_M$ .

Let us express the slip-dependent resistance value  $R_r(S)$  in terms of its maximum value  $R_M$  and substitute it into the heat source equation:

$$W = I^2 \cdot r = I^2 \cdot \frac{R_r(S)}{R_M} \quad (\text{pu W}) \quad (11)$$

Breaking (11) down into positive- and negative-sequence components accommodates motor heating due to balanced current heating (positive sequence) and heating due to any current unbalance that might be present (negative sequence).

$$W_{\text{TOTAL}} = I_1^2 \cdot \frac{R_{r1}(S)}{R_M} + I_2^2 \cdot \frac{R_{r2}(S)}{R_M} \quad (\text{pu W}) \quad (12)$$

Replacing the heat source of Fig. 2 with (12) gives rise to the slip-dependent thermal model shown in Fig. 8.

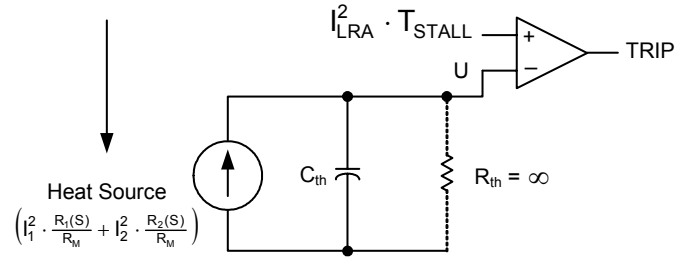


Fig. 8 Slip-Dependent Starting Element

The response curve for this modified starting element is shown in Fig. 9.

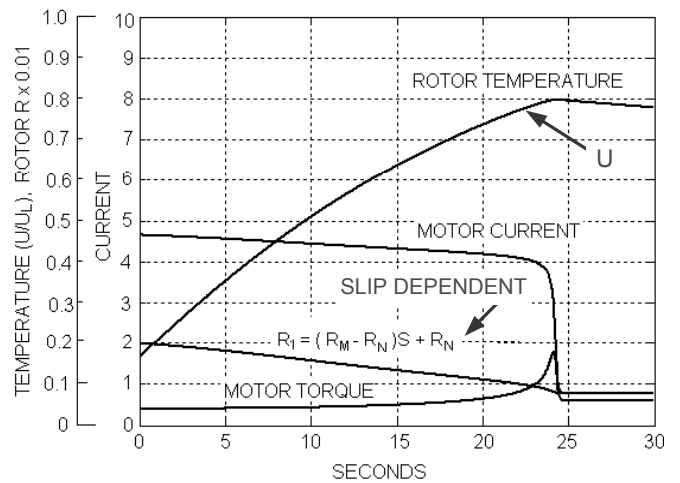


Fig. 9 Slip-Dependent Starting Element Response Curve

Comparison of the standard starting element response curve to the slip-dependent starting element response curve is shown in Fig. 10. It clearly shows that because of the decreasing rotor resistance as a motor accelerates, rotor temperature is not a linear relationship, resulting in the ability to facilitate high-inertia starts without premature motor trips.

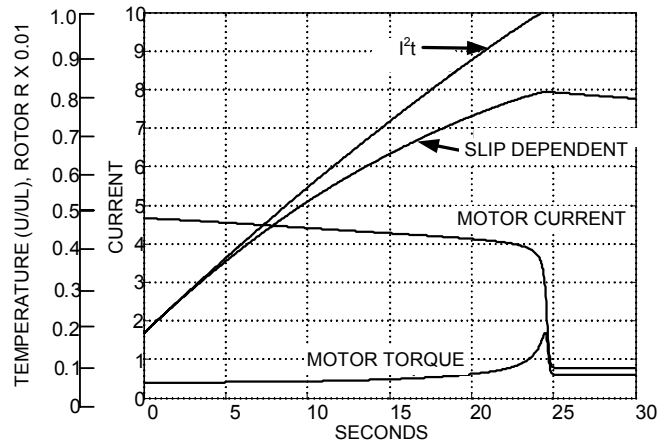


Fig. 10 Comparison of Starting Elements Response Curves

## VIII. FIELD REPORTS

A field case was selected to demonstrate the slip-dependent model effectiveness.

The case motor is a 1,000-hp, 4-kV induction machine. The motor electrical parameters are:

- SF = 1.15
- FLA = 126 A
- LRA = 6 • FLA
- T<sub>STALL</sub> = 10 seconds
- FLS = 0.0055 pu (1,790 rpm)
- LRQ = 0.8 pu

Motor starts are captured by the motor protection relay for a fixed period of sixty seconds at a programmable sampling rate from 0.25 to 5 cycles. The last five motor start reports (MSR) are saved in nonvolatile memory. A 5-cycle rate was used in these reports. MSR reports record the four currents, three voltages, motor slip, and rotor thermal capacity used. The length of the MSR report shown in Fig. 11 was shortened.

Fig. 11 shows the MSR data from stand still up to rated speed.

```
msr
□
HI INERTIA                               Date: 06/02/2006   Time: 10:16:02
MEB:20060531                             Time Source: Internal

FID XXX-XXX-X133-W0-2001001-D20060531

Start Date 06/02/2006
Start Time 10:14:37.345

# Starts          7
Start Time (s)   20.5
Start TCU (%)    79
MaxCurrent (A)   614
MinVoltage (V)   3308
```

CYCLE	IA (A)	IB (A)	IC (A)	IN (A)	VAB (V)	VBC (V)	VCA (V)	TCURTR (%)	SLIP (%)
5.00	612	614	613	0	3311	3308	3313	16.9	100.0
60.00	609	610	610	0	3322	3315	3320	21.8	94.9
120.00	605	606	605	0	3328	3322	3326	26.9	93.3
180.00	601	602	600	0	3333	3327	3330	31.7	87.5
240.00	598	598	597	0	3341	3336	3340	36.3	82.3
300.00	590	592	591	0	3337	3336	3340	40.7	77.8
360.00	586	587	587	0	3349	3344	3351	44.7	70.0
420.00	582	583	582	0	3359	3353	3360	48.5	66.7
480.00	577	577	577	0	3357	3354	3360	52.0	60.9
540.00	571	572	571	0	3367	3363	3369	55.3	55.4
600.00	568	568	567	0	3376	3373	3378	58.4	50.0
660.00	563	564	562	0	3379	3373	3380	61.3	47.0
720.00	559	560	558	0	3385	3378	3386	64.0	41.8
780.00	555	555	554	0	3395	3391	3397	66.5	37.3
840.00	552	552	552	0	3398	3395	3399	68.7	34.1
900.00	546	547	546	0	3402	3399	3405	70.8	29.6
960.00	541	543	541	0	3407	3405	3411	72.9	25.9
1020.00	537	537	535	0	3418	3412	3418	74.7	21.5
1080.00	528	530	529	0	3432	3430	3433	76.3	17.0
1140.00	518	519	518	0	3464	3462	3465	77.8	12.0
1200.00	405	404	406	0	3695	3691	3699	78.9	3.5
1260.00	97	98	97	0	4020	4018	4022	79.0	0.5
1320.00	96	98	96	0	4021	4021	4024	78.7	0.5

Fig. 11 1,000-hp MSR

Fig. 12 is a pictorial representation of Fig. 11, where Imotor is the average of IA, IB, and IC, and TCURTR is the rotor thermal capacity. Fig. 11 illustrates that a machine with a safe stall time of 10 seconds is successfully started in 21 seconds without exceeding the rotor design temperature limit. Only 79% of the temperature limit was reached.

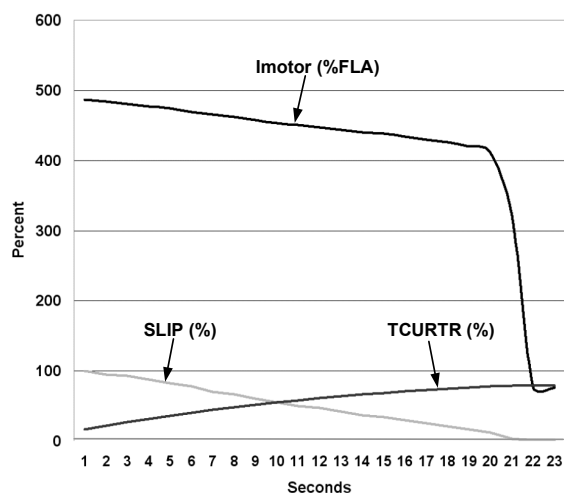


Fig. 12 1,000-hp Pictorial MSR

## IX. CONCLUSIONS

1. The slip-dependent thermal model tracks the motor temperature more accurately than the  $I^2t$  model, thus facilitating high-inertia starts without the use of speed switches.
2. ALWAYS check with the motor manufacturer to ensure that the motor in question is suited for a high-inertia starting application.
3. Ensure that the combined inertia of the rotor/shaft assembly AND the connected load is within motor acceleration capability.

## X. REFERENCES

- [1] Stanley E. Zocholl, "Optimizing Motor Thermal Models," in *IEEE PCIC Conference Record* 2006-29.

## XI. VITA

**Edward A. Lebenhaft** received his B.A.Sc. in Electrical Engineering from the University of Toronto in 1972. He spent 18 years with Ontario Hydro constructing and designing nuclear power plants. In the following 14 years Ed was a regional manager for Multilin (eventually to be bought out by GE). After a brief retirement, Ed joined Schweitzer Engineering Laboratories, Inc. in October 2004, where he is currently a field application engineer dealing with motor protection. Ed is a registered Professional Engineer in South Carolina.