First Order Thermal Model Applied to Cyclic Loads

Ed Lebenhaft
Schweitzer Engineering Laboratories, Inc.

Presented at the
IEEE Cement Industry Technical Conference
Charleston, South Carolina
April 29–May 2, 2007

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By:
IEEE-IAS Cement Industry Committee
Ed Lebenhaft, P.E., Schweitzer Engineering Laboratories, Inc.

Abstract:

This paper provides an overview of the first order thermal model and discusses how to apply this model to motor protection in starting and running state. This paper also demonstrates the first order thermal model's response to cyclic load.

Introduction:

Many industry segments have processes that are categorized as cyclic. Typical examples include the following:

- Crushers
- Pulverizers
- Positive displacement compressors
- Chippers
- Some conveyer belts

If the relay thermal model protecting the motor of a specific cyclic process is inadequately designed, it will eventually cause a nuisance motor trip, disrupting the process. This is not an acceptable condition.

This paper will demonstrate that the first order thermal model is superior for protecting motors with cyclic loads.

Overview of the First Order Thermal Model:

Fig. 1 illustrates the first order thermal model. The major components of the model are as follows:

- Heat Source: Heat flow from the source is I^2r watts (J/s).
- Thermal Capacitance (C_{th}): Thermal capacitance represents a motor that has the capacity (C_{th}) to absorb heat from the heat source. The unit of thermal capacitance is J/°C.
- Thermal Resistance (R_{th}): Thermal resistance represents the heat dissipated by a motor to its surroundings. The unit of thermal resistance is °C/W.
- Comparator: The comparator's function is to compare the calculated motor per unit temperature with a preset value based on the motor manufacturer's data.

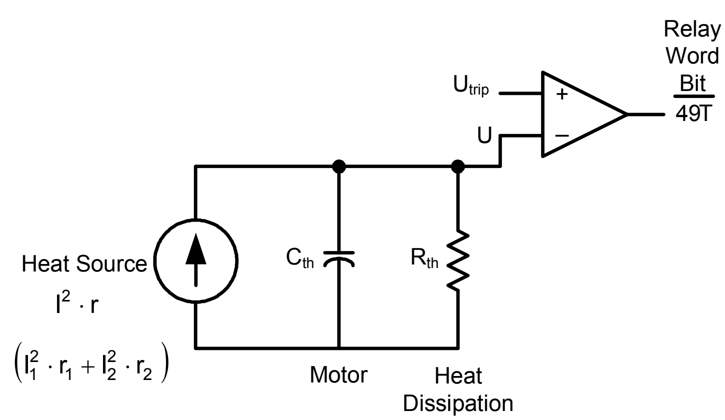

Fig. 1. First Order Thermal Model

For simplicity sake, the heat source in Fig. 1 and subsequent figures and equations is represented by I^2r. In reality, this term consists of two parts—heat flow due to a balanced set of current phasors (positive-sequence currents) and heat flow due to unbalance in current phasors (negative-sequence currents).

The terms are defined as:

I_1 = Motor positive-sequence current
r_1 = Motor positive-sequence resistance
I_2 = Motor negative-sequence current
r_2 = Motor negative-sequence resistance

Thus the complete expression for I^2r is:

$$I^2r = I_1^2r_1 + I_2^2r_2$$

Qualitative analysis of this model states that heat produced by the heat source is transferred to the motor, which, in turn, dissipates the heat to the surrounding environment.

Quantitative analysis is defined by a first order linear differential equation similar to a parallel RC electrical circuit and is:

$$I^2r = C_{th} \cdot \frac{dU}{dt} + \frac{U}{R_{th}} \qquad \text{(W)} \qquad (1)$$

A further step was taken in refining the above thermal model.

Motors are comprised of two major components—stator and rotor.

The stator component's function is to produce a rotating magnetic field (at line frequency) in the air gap and induce voltage in the rotor bars that produces current flow in those bars.

The rotor current produces a magnetic field of its own. The rotor magnetic field is at 90° to the air-gap magnetic field, thus generating torque tangential to the rotor surface and producing rotational force, which turns the shaft.

Because the construction of the stator and rotor is different, so is their thermal characteristic. To accommodate this major difference in stator and rotor thermal properties, the first order thermal model was refined into the following two elements:

- The starting element protects the rotor during the starting sequence.
- The running element protects the stator when the motor is up to speed and running.

Transition from one element to the other was set at 2.5 times the rated full-load current of the motor.

The high-inertia starting solution uses the slip-dependent thermal model and affects the starting element design only. The detailed design of the slip-dependent thermal model is not discussed in this paper.

Applying First Order Thermal Model to Motor Starting (Rotor Protection):

Even though this paper deals with motors in the running state, starting is discussed briefly for completeness.

It is universally accepted that the starting sequence of an ac motor is regarded as an adiabatic (loss less) process. Starting deposits an immense amount of heat (up to a hundred times the rated heating) in rotor bars. Also, the duration of the starting sequence is magnitudes shorter than motor thermal time constants.

Thus it is regarded that any heat deposited in the rotor will not dissipate to the surroundings during the starting sequence (it will dissipate later when the motor is up to speed and running).

If we apply this assumption to the first order thermal model depicted in Fig. 1, we are effectively saying that the thermal resistance of the motor during starting is infinity ($R = \infty$).

Substituting this condition into (1) yields:

$$I^2 \cdot r = C_{th} \cdot \frac{dU}{dt} + \frac{U}{\infty} = C_{th} \cdot \frac{dU}{dt} \qquad \text{(W)} \qquad (2)$$

Rearranging (2):

$$dU = \frac{I^2 \cdot r}{C_{th}} \cdot dt \qquad (^\circ\text{C})$$

Convert the above expression to per unit quantity by substituting the following:

$$r = C_{th} = 1\,\text{pu}$$

Thus:

$$dU = I^2 \cdot dt \qquad (\text{pu } ^\circ\text{C})$$

Integrating the expression:

$$\int dU = \int I^2 \cdot dt = I^2 \int dt$$

The solution to this general integral is:

$$U = I^2 \cdot t \qquad (\text{pu } ^\circ\text{C}) \qquad (3)$$

Motor manufacturers supply rotor thermal limit information as part of motor data. The rotor thermal limit is expressed in terms of the maximum time (T_{STALL}) that corresponding locked-rotor current (I_{LRA}) can be applied to a motor.

Applying this to (3):

$$I = I_{LRA} \text{ (per unit locked-rotor amperes)}$$

$$t = T_{STALL} \text{ (safe stall time, s)}$$

$$U = I_{LRA}^2 \cdot T_{STALL} \qquad (\text{pu } ^\circ\text{C}) \qquad (4)$$

Incorporating all the above changes to Fig. 1 results in the starting element of the first order thermal model as illustrated in Fig. 2.

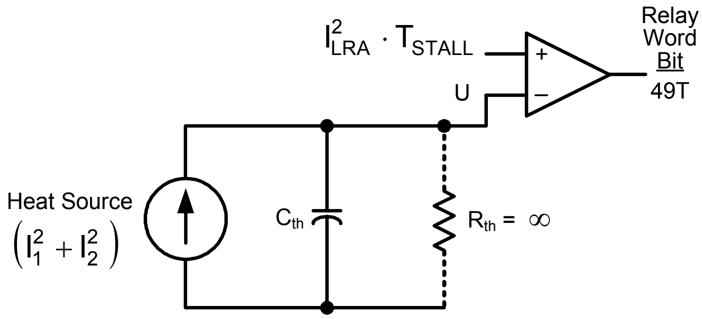

Fig. 2. Starting Element

Applying First Order Thermal Model to Running Motor (Stator Protection):

Once the motor reaches full speed and the current drops below 2.5 • FLA, the first order thermal model switches from starting element to running element.

Equation (1) and Fig. 1 apply to the running element.

To see the response of the running element to current flow, let us solve (1):

Rearrange the terms in (1):

$$I^2 \cdot r \cdot R_{th} = C_{th} \cdot R_{th} \cdot \frac{dU}{dt} + U \qquad (^\circ C) \qquad (5)$$

The above equation can be further simplified by converting it to a per unit quantity. The base quantities are:

Current = I_{FLA} (full-load current)

Temperature at rated current = $I_{FLA}^2 \bullet r \bullet R_{th}$

Convert (5) to a per unit quantity:

$$\frac{I^2 \cdot r \cdot R_{th}}{I_{FLA}^2 \cdot r \cdot R_{th}} = I^2 = C_{th} \cdot R_{th} \cdot \frac{dU}{dt} + U \qquad (\text{pu } ^\circ C) \qquad (6)$$

Now substitute motor thermal time constant τ_{th} for C_{th} and R_{th}:

$$\tau = R_{th} \cdot C_{th}$$

$$I^2 = \tau \cdot \frac{dU}{dt} + U \qquad (\text{pu } ^\circ C) \qquad (7)$$

It is extremely important to note that the quantity I^2 represents per unit temperature; in other words, per unit temperature is proportional to the square of the per unit current!

Equation (7) is a first order linear differential equation whose solution is:

$$U(t) = I_0^2 \cdot e^{-\frac{t}{\tau_{th}}} + I^2 \cdot (1 - e^{-\frac{t}{\tau_{th}}}) \qquad (\text{pu } ^\circ C) \qquad (8)$$

Where:

$U(t)$ = Per unit temperature as a function of time
I_0 = Per unit initial current
I = Per unit final current
τ_{th} = Motor running thermal time constant

Another useful presentation of (8) to motor relay engineers is the time (t) in which the running element will reach temperature U(t).

Rewriting (8) yields:

$$t = \tau \cdot \ln\left[\frac{I^2 - I_0^2}{I^2 - U(t)}\right] \qquad \text{(s)} \qquad (9)$$

In plain language, (9) states that the time it takes to reach per unit temperature, U(t), is calculated by multiplying the thermal time constant of the motor by the natural logarithm of the difference between final per unit temperature and initial per unit temperature, divided by the difference between final per unit temperature and the per unit temperature, U(t), in question.

Two important reminders are:

- The base for this per unit system is motor full-load current (FLA).
- A valid range for per unit temperature, U(t), is anywhere between initial per unit temperature, I_0^2, and final per unit temperature, I^2.

Let us further simplify (9) to make it more suitable for motor protection applications.

Manufacturers state the service factor (SF) of the machine on every motor nameplate. Even though the exact interpretation of the SF is vague, one thing is certain—any motor current greater than SF • FLA is considered a running overload condition. Translate this into a maximum per unit temperature that the motor is designed for and can sustain at any time (t):

$$U(t) = (SF \cdot I_{FLA})^2 \qquad \text{(pu °C)} \qquad (10)$$

Because I_{FLA} = 1 per unit, the above expression is further simplified to:

$$U(t) = SF^2 \qquad \text{(pu °C)} \qquad (11)$$

Substituting (11) into (9) results in the final equation of the first order thermal model running element:

$$t = \tau \cdot \ln\left[\frac{I^2 - I_0^2}{I^2 - SF^2}\right] \qquad \text{(s)} \qquad (12)$$

A closer examination of (12) reveals that in order for t to be a rational number, the overload current, I, has to be greater than SF • I_{FLA}.

The question then is: what happens in a case where I is less than SF • I_{FLA}?

This situation does not constitute an overload condition, but rather reflects the element's response to a load change.

Let us resort back to (9). Because the denominator of (9) has to be greater than zero to maintain t, a rational number, let us postulate (using the three time constants rule, which states that it takes three time constants for a quantity to exponentially decay to 5% of its original) that once the per unit temperature U(t) reaches within 5% (0.05 per unit) of the final per unit temperature I^2, the equation is satisfied, i.e.:

$$I^2 - U(t) = 0.05 \qquad \text{(pu °C)} \qquad (13)$$

Substituting (13) into (9) and using absolute values to account for increasing or decreasing load yields:

$$t = \tau \cdot \ln\left[\frac{\left|I^2 - I_0^2\right|}{0.05}\right] = \tau \cdot \ln\left(20 \cdot \left|I^2 - I_0^2\right|\right) \qquad \text{(s)} \qquad (14)$$

Equations (12) and (14) represent the running element dynamic response to any current magnitude.

<u>Running Element Response to Cyclic Load:</u>

Cyclic loads are defined as loads varying from one load level to another load level at constant periodic intervals.

In terms of motor current, cyclic load represents motor currents that fluctuate from one value to another and then back to the original value at constant intervals.

Equipped with the running model described by (12) and (14), we can now proceed and apply the model to a cyclic load.

Two case motors were chosen: case motor #1 has an SF equal to 1.0 and a running thermal time constant (RTC) equal to 1,200 seconds, as shown in Figs. 3 and 4, and case motor #2 has an SF equal to 1.15 and an RTC equal to 1,200 seconds, as shown in Figs. 5 and 6.

The cyclic currents were selected to exceed the service factors, yet still be realistic.

For each case motor two conditions are shown:

- The cyclic current period is 600 seconds (Figs. 3 and 5).
- The cyclic current period is 1,200 seconds (Figs. 4 and 6).

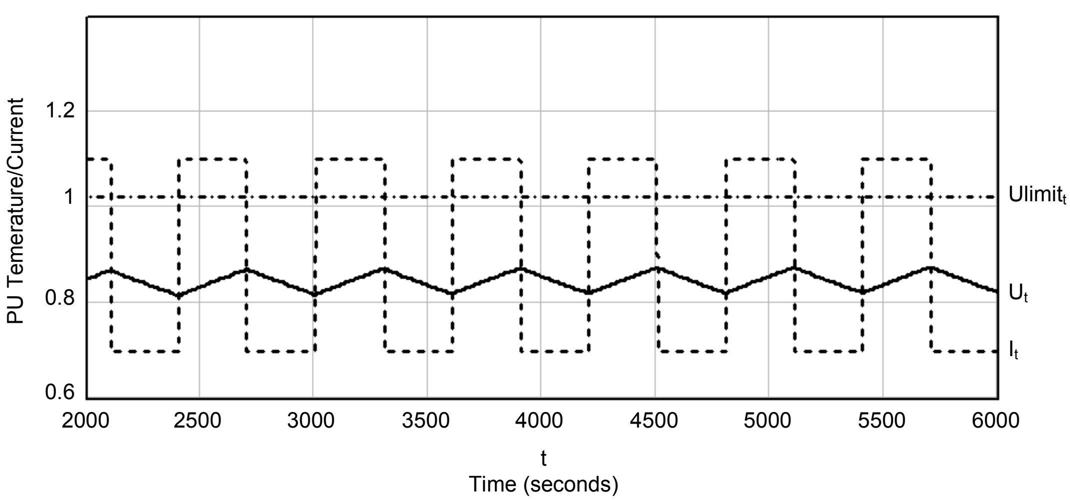

Fig. 3. Response to Cyclic Load: SF Equal to 1.01 and Current Period Equal to 600 Seconds

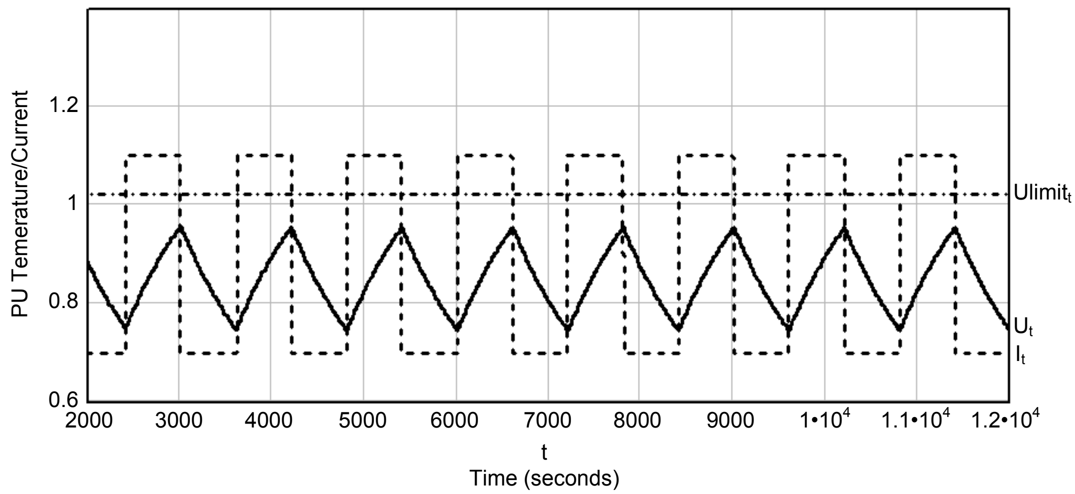

Fig. 4. Response to Cyclic Load: SF Equal to 1.01 and Current Period Equal to 1,200 Seconds

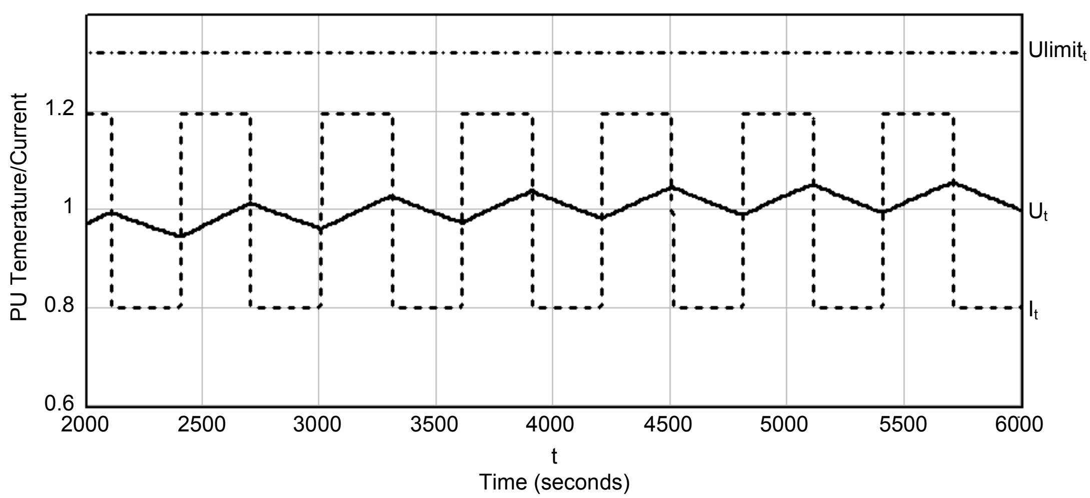

Fig. 5. Response to Cyclic Load: SF Equal to 1.15 and Current Period Equal to 600 Seconds

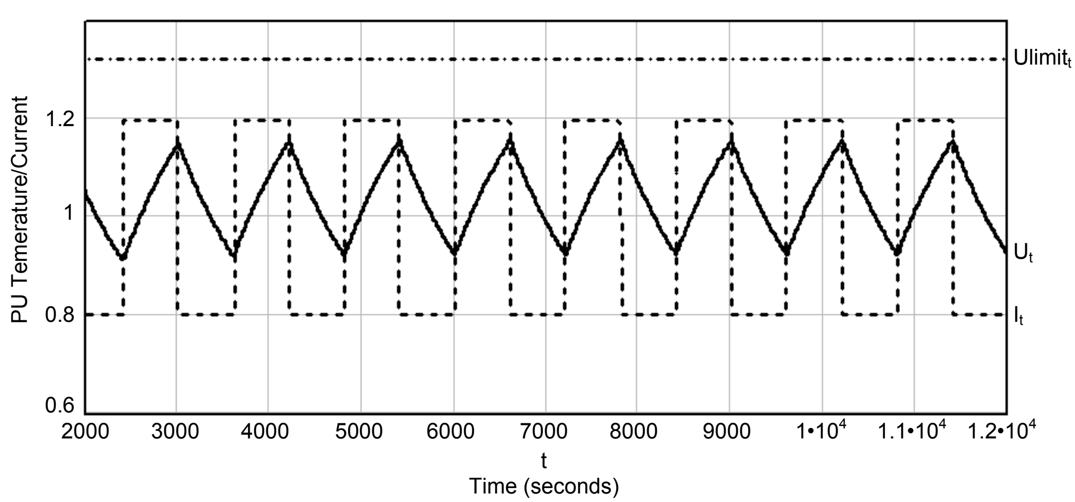

Fig. 6. Response to Cyclic Load: SF Equal to 1.15 and Current Period Equal to 1,200 Seconds

Conclusions:

As demonstrated, the first order thermal model faithfully tracks cyclic load without a premature process interruption due to motor trip, even though the maximum current exceeds the SF rating, as long as the motor pu temperature stays below the design limit of SF^2.

As the period of the cyclic current approaches the motor RTC and the maximum current exceeds the SF rating, the motor temperature will approach the motor design limit pu temperature and will result in motor trip and process interruption.

It is imperative that when applying a motor to a cyclic load application the motor thermal, as well as the electrical characteristics need to be scrutinized and compared to the mechanical process characteristics to ensure compatibility.

Reference:

[1] Stanley E. Zocholl, “Optimizing Motor Thermal Models,” IEEE PCIC Conference Record 2006-29.

Biography:

Edward A. Lebenhaft received his B.A.Sc. in Electrical Engineering from the University of Toronto in 1972. He spent 18 years with Ontario Hydro constructing and designing nuclear power plants. In the following 14 years, Ed was a regional manager for SE USA with Multilin (eventually to be bought out by GE). After a brief retirement, Ed joined Schweitzer Engineering Laboratories, Inc. in October 2004 where he is currently a field application engineer dealing with motor protection. Ed is a registered Professional Engineer in South Carolina.

Previously presented at the IEEE Cement Industry Technical Conference, Charleston, SC, April 2007.

20070215 • TP6267-01