# Man-Made Faults - Line Protection Operation for an Unintended Phase Cross-Connect Condition 

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# Man-Made Faults - Line Protection Operation for an Unintended Phase Cross-Connect Condition 

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#### Abstract

For a temporary operating condition, a utility installed a new switch to tie together two subtransmission lines. During the installation, operating personnel inadvertently crossed two phases. When the operators closed the line switch, it created an unusual fault, cross-connecting two phases between the sources and causing the line protection to operate on one end of the line. Another relay protecting a line connected to a common bus also tripped for this condition.

The elements that operated in these microprocessor-based relays were compensator phase distance elements. Compensator phase distance relays have evolved from the electromechanical relays of the 1950s to protection elements included in today's microprocessor-based relays. In this paper, we discuss the theory of operation of compensator phase distance relays and some of their applications. We then explain why the compensator phase distance elements operated for this phase cross-connect condition by analyzing the event reports from the relays involved.

We use symmetrical components to demonstrate how to analyze the phase cross-connect condition and analyze the responses to this condition by a negative-sequence voltage-polarized directional element and a positive-sequence polarized mho element. Finally, we discuss methods for making a compensator phase distance element secure for similar phase cross-connect conditions.


## I. Introduction

A utility operates a network of 34 kV lines in their subtransmission system. Because many of these lines are part of a looped system with sources at both ends, the utility applies overcurrent relays torque-controlled by phase distance relays for phase fault protection. The phase distance relays provide directional control of the phase overcurrent relays and also allow for increased relay loadability on the lines. Directional ground overcurrent relays are applied for ground fault protection.

The utility installed a line switch connecting two radial 34 kV lines at a location remote from the substation. This was done to tie the two 34 kV buses together during an outage of the bus tie breaker. A one-line diagram of the system is shown in Fig. 1.

During installation of the line switch, operating personnel inadvertently crossed Phases A and B across the switch. Phasing across the switch was not checked. When the operators closed the line switch, it created an unusual fault, cross-connecting two phases between the sources. Relay 2 and Relay 3 operated for the phase cross-connect fault. Relay 1 did not operate for this event.


Fig. 1. One-Line Diagram of System
The utility originally questioned why Relay 3 , which was protecting a radial line isolated from the cross-connected switch, operated for this event. The authors of this paper further questioned why Relay 2 on the affected line operated, but Relay 1 on the affected line did not. This led to much analysis of how all three relays responded. All three relays are microprocessor-based relays that apply compensator phase distance elements. In order to understand why each compensator distance element responded the way it did for this event, it is beneficial to review the concepts of phase compensator distance relays.

## II. Compensator Distance Elements

The electromechanical compensator distance relay was first introduced in the 1950s. Reference [1] details the theory behind the compensator distance relaying principle. A single relay contains two polyphase distance elements: a phase-tophase element, which responds to all phase-to-phase faults, and a three-phase element, which responds to three-phase
faults. Either of these elements will respond to phase-to-phase-to-ground faults. The ability to apply one polyphase relay to detect all types of phase faults was innovative at the time.

The electromechanical relay design uses compensators to provide the two inputs ( A and B ) to an induction cylinder element. Compensators are air-gap transformers that are energized with current on the primary windings and that provide voltage proportional to the current on the secondary windings. The current energizing the primary winding comes from the relaying current transformers (CTs). The current-derived secondary voltages are subtracted from the system voltage at the relay location. The two compensated voltages are applied to the inputs of the induction cylinder element. The induction cylinder is basically a polyphase induction motor. Fig. 2 shows an induction cylinder element [2].


Fig. 2. Induction Cylinder Relay
The applied compensated voltages ( A and B ) develop either a closing or opening torque, depending on the fault type and location. The design of the compensator elements is different for the phase-to-phase and the three-phase elements. In the phase-to-phase element, three compensators apply compensated phase-to-phase voltages to the two induction cylinder inputs. Operating torque occurs when the phase rotation on the applied compensated voltages reverses. In the three-phase element, one compensator is applied. See [1] for more details.

The compensator distance element was later implemented in solid-state relays using angle comparators with coincidence timers.

Compensator distance elements were implemented in the early microprocessor-based relays. The compensators used in the electromechanical relays were replaced with mathematical models used in the calculations performed by the microprocessor-based relays. Reference [3] provides (1)
and (2), which are used to implement the phase-to-phase element in one microprocessor-based relay.

$$
\begin{align*}
\mathrm{A} & =\mathrm{V}_{\mathrm{AB}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{\mathrm{AB}} \\
\mathrm{~B} & =\mathrm{V}_{\mathrm{BC}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{\mathrm{BC}}  \tag{1}\\
\mathrm{~T} & =\operatorname{Im}\left[\mathrm{A} \cdot(\mathrm{~B})^{*}\right] \tag{2}
\end{align*}
$$

The compensator distance element torque is the imaginary part of the product between Phasor $A$ and the complex conjugate of Phasor B (the complex conjugate is indicated by an asterisk [*] above the expression). The element operates for negative torque and restrains for positive torque. $\mathrm{V}_{\mathrm{AB}}$ and $\mathrm{V}_{\mathrm{BC}}$ are the measured line voltages, $\mathrm{I}_{\mathrm{AB}}$ and $\mathrm{I}_{\mathrm{BC}}$ are measured currents, and $Z_{1 R}$ is the reach setting in secondary ohms as measured by the relay.

$$
\begin{equation*}
\mathrm{Z}_{1 \mathrm{R}}=\mathrm{r} \cdot \mathrm{Z}_{1} \angle \mathrm{MTA} \tag{3}
\end{equation*}
$$

The value of $r$ indicates the per-unit reach needed to operate the element and $\mathrm{Z}_{1} \angle \mathrm{MTA}$ is the positive-sequence line impedance of the protected line at its angle of maximum torque.

Another approach to analyze the response of the phase-to-phase element of the compensator distance element is to use symmetrical components. The compensated voltages (A and B) in (1) can be written in terms of symmetrical components. Note that all voltages in sequence networks are phase-to-neutral quantities. This results in (4) and (5).

$$
\begin{align*}
& \mathrm{V}_{1} \mathrm{C}=\mathrm{V}_{1 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{1 \mathrm{~A}}  \tag{4}\\
& \mathrm{~V}_{2} \mathrm{C}=\mathrm{V}_{2 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{2 \mathrm{~A}} \tag{5}
\end{align*}
$$

where:
$\mathrm{V}_{1 \mathrm{~A}}$ and $\mathrm{V}_{2 \mathrm{~A}}$ are the positive- and negative-sequence voltages, respectively, of the phase-to-neutral voltages measured by the relay at the voltage transformer (VT) location.
$\mathrm{I}_{1 \mathrm{~A}}$ and $\mathrm{I}_{2 \mathrm{~A}}$ are the positive- and negative-sequence currents, respectively, of the current measured by the relay.
$\mathrm{Z}_{1 \mathrm{R}}$ is as defined in (3).
Effectively, $\mathrm{V}_{1} \mathrm{C}$ and $\mathrm{V}_{2} \mathrm{C}$ are the compensated positiveand negative-sequence voltages at the relay set point.

The balance point, where the calculated torque in (2) equals zero, is defined by (6), where $\theta$ can be any angle.

$$
\begin{equation*}
\frac{\mathrm{V}_{2} \mathrm{C}}{\mathrm{~V}_{1} \mathrm{C}}=1 \angle \theta \tag{6}
\end{equation*}
$$

The phase-to-phase element of the compensator distance element operates when the magnitude of the ratio in (6) is greater than one.

$$
\begin{equation*}
\frac{\left|\mathrm{V}_{2} \mathrm{C}\right|}{\left|\mathrm{V}_{1} \mathrm{C}\right|}>1 \tag{7}
\end{equation*}
$$

The condition in (7) represents a phase reversal of the compensated voltages as calculated in (1). In this condition, the induction cylinder in Fig. 2 rotates in the clockwise direction.

Consider the example system shown in Fig. 3 for an illustration of the concept presented in (7). The left source, right source, and line each have an impedance of 1 per unit (pu). A compensator distance relay is physically located at Bus 1 with a reach set equal to 1 pu to see faults up to Bus 2 .


Fig. 3. Example System for Compensator Relay Evaluation
Fig. 4 illustrates the positive- and negative-sequence voltage magnitudes at Bus 1 (a) and Bus 2 (b), the compensated positive- and negative-sequence voltage magnitudes at the relay set point (c), and the torque evaluation (d) from (2) for all fault locations in the example system from Fig. 3.


Fig. 4. Compensator Relay Evaluation for Phase-to-Phase Fault
For phase-to-phase faults behind the relay, the fault current is flowing in the reverse direction ( $\mathrm{I}_{1}$ is negative and $\mathrm{I}_{2}$ is positive), which causes $\left|\mathrm{V}_{1} \mathrm{C}\right|$-as shown in (4)-to become larger than the measured $\left|\mathrm{V}_{1}\right|$ at Bus 1 ; and $\left|\mathrm{V}_{2} \mathrm{C}\right|-$ as shown in (5)-to become smaller than the measured $\left|\mathrm{V}_{2}\right|$ at Bus 1. The result of (4) and (5) for a reverse fault is that the compensated voltages $\left(\left|\mathrm{V}_{1} \mathrm{C}\right|\right.$ and $\left.\left|\mathrm{V}_{2} \mathrm{C}\right|\right)$ are equivalent to the measured $\left|\mathrm{V}_{1}\right|$ and $\left|\mathrm{V}_{2}\right|$ values that would be seen at Bus 2, which produces restraining torque.

When the fault is located in the forward direction, the current direction changes ( $\mathrm{I}_{1}$ is positive and $\mathrm{I}_{2}$ is negative). For a close-in fault at the relay location, the measured $\left|\mathrm{V}_{1}\right|$ and measured $\left|\mathrm{V}_{2}\right|$ at Bus 1 equal 0.5 pu . Recognizing this, along with $\left|\mathrm{I}_{1}\right|=\left|\mathrm{I}_{2}\right|$ for a phase-to-phase fault, it can be seen that $\left|\mathrm{V}_{2} \mathrm{C}\right|$ from (5) will be larger than $\left|\mathrm{V}_{1} \mathrm{C}\right|$ from (4) and the relay
will operate for a close-in fault. As the fault point moves farther away from the relay, $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ move closer to each other until they are equal to 0.5 pu at the relay set point. At this point, no torque is produced in either the positivesequence direction or the negative-sequence direction and the relay is at the balance point defined in (6).

When the fault is located beyond the reach setting of the relay, $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ once again follow the measured voltages at Bus 2 and the relay properly restrains.

It can be seen from Fig. 4 c and Fig. 4 d that the relay generates the largest operation torque for close-in faults $\left(\left|\mathrm{V}_{2} \mathrm{C}\right| \gg\left|\mathrm{V}_{1} \mathrm{C}\right|\right)$. This operation torque declines as the fault location is moved down the line until there is no operation torque at the reach setting. Also, strong restraining torque is produced for reverse faults, making the compensator relay inherently directional.

The three-phase compensator distance element is implemented in one microprocessor-based relay using the same torque equation as in (2) with the following inputs for A and B , where $k$ is a design constant of the relay and $\mathrm{V}_{\mathrm{C}(\mathrm{mem})}$ is the Phase C memory voltage:

$$
\begin{align*}
& \mathrm{A}=\mathrm{V}_{\mathrm{AB}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{\mathrm{AB}} \\
& \mathrm{~B}=-\mathrm{j} \mathrm{~V}_{\mathrm{AB}}-\mathrm{k} \cdot \mathrm{~V}_{\mathrm{C}(\mathrm{mem})} \tag{8}
\end{align*}
$$

The memory voltage is required to allow the element to operate for a bolted fault in front of the relay where all three phase voltages measured by the relay equal zero.

The three-phase compensator distance element implementation used in (8) is significantly different from the electromechanical implementation presented in [1]. However, both the microprocessor-based and electromechanical implementations respond to three-phase and phase-to- phase-to-ground faults. In the microprocessor-based implementation, because only measured voltage is being used for the calculation for Phasor B, the overall result of (2) is a dual-voltage-polarized mho element, because fault loop voltage $\left(\mathrm{V}_{\mathrm{AB}}\right)$ and nonfault voltage $\left(\mathrm{V}_{\mathrm{C}}\right)$ are used for polarization. There will be some expansion in the mho circle back to the source.

Relays with compensator distance elements and relays having phase-pair ( $\mathrm{AB}, \mathrm{BC}$, and CA ) mho elements, polarized with positive-sequence memory voltage, have been applied over the years to protect transmission lines. Some considerations for applying compensator distance relays include the following:

- Desiring a different phase-distance operating principle for backup protection.
- Protecting a transmission line through a delta-wye transformer (see the appendix).
- Using a three-phase element to block reclosing for three-phase faults.
- Protecting a line equipped with open-delta connected VTs.


## III. ANALYSIS OF EvENT

Event reports for the three relays involved in the example fault are analyzed in this section, and the results are revisited in Section V. The behavior of the protection elements enabled in these relays is discussed as well as other common relay elements not implemented in this application. We start our analysis with Relay 1, the relay that did not operate for this fault.

## A. Relay 1 Analysis

Relay 1 uses a compensator distance element to protect the faulted line. The Zone 1 reach in service for Relay 1 at the time of the cross-connect condition was 120 percent of the line impedance $\left(0.268 \angle 57^{\circ}\right.$ ohms secondary). While it is uncommon to overreach a line section with a Zone 1 distance element, the fact that the reach was set in this manner makes it puzzling that the relay restrained for a cross-connect condition within the line section. While the relay did not trip, it did trigger an event report on an instantaneous overcurrent element set at $4,800 \mathrm{~A}$. The maximum phase current magnitude seen during this fault by Relay 1 was $5,368 \mathrm{~A}$. The fault study supplied by the utility indicates that the expected Relay 1 phase current for a phase-to-phase fault at the position of the switch is about $10,000 \mathrm{~A}$. The oscillography for the event from Relay 1 is shown in Fig. 5.


Fig. 5. Relay 1 Event Report Oscillography
Relay 1 sees what appears to be a Phase-A-to-Phase-B fault. We note that the Phase A and Phase B currents are out of phase with each other, and the Phase A and Phase B voltages are depressed. For a forward Phase-A-to-Phase-B fault, $\mathrm{V}_{\mathrm{AB}}$ should lead $\mathrm{I}_{\mathrm{AB}}$ current. Fig. 6 shows the prefault phase-to-ground voltages and currents on the left and the phase-to-phase voltage and current at Cycle 7 on the right.

As expected, the fault appears to be in the forward direction because $\mathrm{V}_{\mathrm{AB}}$ leads $\mathrm{I}_{\mathrm{AB}}$.

As discussed in Section II, we can compare the $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ values to determine the operation of the compensator distance element. If $\left|\mathrm{V}_{2} \mathrm{C}\right|$ is greater than $\left|\mathrm{V}_{1} \mathrm{C}\right|$, the relay operates; otherwise, the element restrains. Fig. 7 shows the $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ values seen by Relay 1 in primary kilovolts.


Fig. 6. Prefault Phasors (left) and Phasors at Cycle 7 (right)


Fig. 7. $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ for Relay 1
As can be seen from Fig. 7, the $\left|\mathrm{V}_{1} \mathrm{C}\right|$ is always larger than $\left|\mathrm{V}_{2} \mathrm{C}\right|$, thus preventing operation.

Clearly, the compensator distance element saw this fault outside its zone of protection. Using an iterative technique to solve for the reach setting that would allow $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ to be equal, we find that a Zone 1 reach setting of approximately 200 percent of the line would have been at the operate point.

We now review how a self-polarized mho element would have performed for this fault. A self-polarized mho element digital implementation for Phase-A-to-Phase-B faults is defined by (9) [2]. While nearly all digital implementations of mho elements are not self-polarized, self-polarized mho elements are more convenient to evaluate than mho elements with memory polarization. In addition, for traditional faults with no fault resistance, the behavior of both elements is very similar. Evaluating a self-polarized mho element gives protection engineers a quick way to determine apparent impedance to the fault.

$$
\begin{equation*}
\mathrm{MAB}=\frac{\operatorname{Re}\left(\mathrm{V}_{\mathrm{AB}} \cdot \overline{\mathrm{~V}_{\mathrm{AB}}}\right)}{\operatorname{Re}\left(1 \angle \mathrm{MTA} \cdot \mathrm{I}_{\mathrm{AB}} \cdot \overline{\mathrm{~V}_{\mathrm{AB}}}\right)} \tag{9}
\end{equation*}
$$

Using the same maximum torque angle (MTA) setting as the compensator distance element $\left(\angle 57^{\circ}\right)$, the self-polarized mho element indicates a reach at 200 percent of the total line impedance ( 0.448 ohms secondary). Because the value returned by (9) is greater than the set point, the element restrains.

In summary, the protection elements reviewed for the location of Relay 1 indicate that the fault is well beyond the remote terminal. This would place the apparent fault in the autotransformer connected to Bus 2, which is behind Relay 2.

## B. Relay 2 Analysis

Next we analyze Relay 2, the line relay that operated for this fault. Relay 2 also uses compensator distance elements to protect the line. The Zone 1 elements are set at 80 percent of
the line impedance $\left(0.253 \angle 52^{\circ}\right.$ ohms secondary). Fig. 8 shows the oscillography from the event.

Relay 2 also sees what appears to be a Phase-A-to-Phase-B fault. The maximum phase current magnitude seen by Relay 2 is $5,580 \mathrm{~A}$, which is very close to the phase current magnitude seen by Relay 1. The fault study supplied by the utility indicates that the expected Relay 2 phase current for a Phase-A-to-Phase-B fault at the position of the switch is about 4,000 A. Relay Word bit 21P, which corresponds to the output of the Zone 1 compensator distance element, asserts and stays asserted throughout the fault. Fig. 8 also shows some unexpected data for a phase-to-phase fault. The measured $\left|\mathrm{V}_{2}\right|$ is larger than the measured $\left|\mathrm{V}_{1}\right|$ during the fault.


Fig. 8. Relay 2 Event Report Oscillography
A traditional phase-to-phase fault will produce the largest magnitude of $\left|\mathrm{V}_{2}\right|$ at the point of fault. At this point, $\left|\mathrm{V}_{1}\right|=\left|\mathrm{V}_{2}\right|=0.5 \mathrm{pu}$. As you move away from the fault point, $\left|\mathrm{V}_{1}\right|$ will increase and $\left|\mathrm{V}_{2}\right|$ will decrease. $\left|\mathrm{V}_{2}\right|$ will never be larger than $\left|\mathrm{V}_{1}\right|$ at any point on the system.

To see how $\left|\mathrm{V}_{2}\right|$ greater than $\left|\mathrm{V}_{1}\right|$ affects the phase voltages, we look at the prefault and fault phase voltages for Relay 2 in Fig. 9.


Fig. 9. Relay 2 Prefault Phase Voltages (left) and Fault Phase Voltages (right)

In Fig. 9, the unfaulted phase $\left(\mathrm{V}_{\mathrm{C}}\right)$ is the reference. By convention, phasors rotate in the counterclockwise direction. In the left-hand panel of Fig. 9, if there were a stationary observer at 0 degrees and this observer were watching phasors
pass, he would see ABC phase rotation. However, in the righthand panel of Fig. 9, an observer at 0 degrees would see ACB phase rotation. When the magnitude of $\left|\mathrm{V}_{2}\right|$ is larger than the magnitude of $\left|\mathrm{V}_{1}\right|$, the phase rotation reverses.

Because the phase rotation changed unexpectedly during this event, we decided to evaluate the negative-sequence impedance directional element $\left(Z_{2}\right)$ to determine the directional evaluation of this element. Reference [4] and (10) define the $Z_{2}$ element that is available in current state-of-theart relays. While the relay in service at the time of this fault did not have a directional element to supervise the compensator element, we can quickly see how it performs to help understand the fault.

$$
\begin{equation*}
\mathrm{Z}_{2}=\frac{\operatorname{Re}\left[\mathrm{V}_{2} \cdot \overline{\left(\mathrm{I}_{2} \cdot 1 \angle \mathrm{MTA}\right)}\right]}{\left|\mathrm{I}_{2}\right|^{2}} \tag{10}
\end{equation*}
$$

The $Z_{2}$ element will evaluate to a negative number for forward faults and a positive number for reverse faults. Additionally, because the element produces a magnitude, the user may select directional setting thresholds to alter the sensitivity, security, and dependability of the element. For the purpose of our analysis, we simply look at the sign generated by the numerator and disregard the magnitude of the $\mathrm{Z}_{2}$ output.

With this in mind, the numerator in (10) can be rewritten as (11).

$$
\begin{equation*}
\mathrm{Z}_{2 \mathrm{~N}}=\cos \left(\angle \mathrm{V}_{2}-\angle \mathrm{I}_{2} \mathrm{C}\right) \tag{11}
\end{equation*}
$$

where:

$$
\angle \mathrm{I}_{2} \mathrm{C}=\angle \mathrm{I}_{2}+\mathrm{MTA}
$$

The cosine function evaluates to a negative number for angles greater than 90 degrees and less than -90 degrees and a positive number for angles less than 90 degrees and greater than -90 degrees. Using this information, we can construct "blinders" at +90 degrees and -90 degrees from $V_{2}$ and see where $\mathrm{I}_{2} \mathrm{C}$ plots in a phasor view. The results of this are shown in Fig. 10.


Fig. 10. Sign Evaluation of $\mathrm{Z}_{2}$ Element for Relay 2
Fig. 10 shows $\mathrm{I}_{2} \mathrm{C}$ left of Blinder 1 and Blinder 2, which means (11) evaluates to a negative number and the $Z_{2}$ element will declare a forward fault. This agrees with the position of the switch. Also, it shows that even though $\left|\mathrm{V}_{2}\right|$ is larger than $\left|V_{1}\right|$ for this fault, the $Z_{2}$ element appears to point in the direction of the switch (forward).

Next we evaluate the compensator distance element in Fig. 11 by comparing $\left|\mathrm{V}_{1} \mathrm{C}\right|$ to $\left|\mathrm{V}_{2} \mathrm{C}\right|$.


Fig. 11. $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ for Relay 2
$\left|\mathrm{V}_{2} \mathrm{C}\right|$ is larger than $\left|\mathrm{V}_{1} \mathrm{C}\right|$, so the element operates. In fact, the relay would have operated under the entire range of set points allowed for Zone 1 reach in this relay. Because we know that current flows in the forward direction based on the evaluation of the $\mathrm{Z}_{2}$ element, $\left|\mathrm{V}_{2} \mathrm{C}\right|$ will be larger than the relay-measured $\left|\mathrm{V}_{2}\right|$. Also, $\left|\mathrm{V}_{1} \mathrm{C}\right|$ will be smaller than the relaymeasured $\left|\mathrm{V}_{1}\right|$. In effect, the apparent phase reversal becomes more pronounced in the compensated voltages.

Finally, we review the behavior of the self-polarized mho element defined in (9). Using the same MTA setting as the compensator distance element $\left(\angle 52^{\circ}\right)$, the self-polarized mho element indicates a reach at -114 percent of the total line impedance ( -0.36 ohms secondary). This is less than the set point of 0.253 ohms, but the relay does not operate. This is because the digital implementation of the self-polarized element is not simply a comparator that asserts if the measured impedance is less than the set point. There must be additional supervision to prevent operation for reverse faults. In an electrometrical implementation, a reverse out-of-zone fault produces restraining torque. While it is clear that the digital implementation restrains for a forward out-of-zone fault, we need to discuss how it restrains for a reverse out-of-zone fault.

The sign of the impedance evaluation comes from the denominator of (9). Essentially, the denominator is a built-in directional element for the mho element. Using the same technique as for evaluating the sign of the $\mathrm{Z}_{2}$ element, we can evaluate the directionality of the mho element. Equation (12) is the denominator of (9) rewritten to obtain the sign.

$$
\begin{equation*}
\mathrm{MABD}=\cos \left(\angle \mathrm{I}_{\mathrm{AB}} \mathrm{C}-\angle \mathrm{V}_{\mathrm{AB}}\right) \tag{12}
\end{equation*}
$$

where:

$$
\angle \mathrm{I}_{\mathrm{AB}} \mathrm{C}=\angle \mathrm{I}_{\mathrm{AB}}+\mathrm{MTA}
$$

Note that the sign convention for forward and reverse is different in the self-polarized mho element as compared with the $Z_{2}$ element. In the self-polarized mho element, a negative value is a reverse fault while a positive number is a forward fault.

As seen in Fig. 12, $\mathrm{I}_{\mathrm{AB}} \mathrm{C}$ plots in the reverse direction, meaning the denominator evaluates to a negative number. If the denominator of the self-polarized mho element is negative, the element is prevented from operating.


Fig. 12. Mho Element Directional Evaluation for Relay 2
A digital implementation of a self-polarized mho element that operates identically to an electromechanical implementation is additionally supervised by the denominator term, as shown in Fig. 13.


Fig. 13. Possible Digital Implementation of a Self-Polarized Mho Element
In digital relays, many types of supervising quantities can be chosen. Another option to restrain for reverse faults is to use dedicated directional elements, such as the $Z_{2}$ element described in (10). This can be done in addition to or in lieu of the MABD term. It can be seen that if $Z_{2}$ had been chosen as the directional supervision instead of MABD, the selfpolarized element would have operated because the apparent impedance was less than the Zone 1 reach setting and the $\mathrm{Z}_{2}$ element declared forward. However, in the remainder of this paper, we will evaluate the mho elements using Fig. 13 and conclude that a self-polarized element would not have operated for this fault.

There are two interesting things to note about the $Z_{2}$ element and the self-polarized mho element for Relay 2:

- The $Z_{2}$ element directionality disagrees with the mho element directionality at Relay 2.
- The mho element evaluation indicates a reverse fault at Relay 2. Interestingly, this agrees with the mho element at Relay 1.


## C. Summary of Relay 1 and Relay 2 Operation

Based on the analyses performed on Relay 1 and Relay 2, we can see that this cross-connect condition does not appear to be equivalent to a phase-to-phase fault at the location of the switch. Table I shows a summary of the element operation. Note that the Relay 2 compensator distance element operates due to phase reversal.

TABLE I
Summary of Relay 1 and Relay 2 Element Performance

| $\mathbf{Z}_{\mathbf{2}}$ |  | Mho |  | Compensator |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Relay 2 | Relay 1 | Relay 2 | Relay 1 | Relay 2 | Relay 1 |
| Forward | Forward | -0.36 <br> ohms <br> secondary | 0.448 <br> ohms <br> secondary | N/A | 0.448 <br> ohms <br> secondary |
| Fault on line |  | Fault behind <br> Relay 2 |  | Fault behind <br> Relay 2 |  |

The following summary statements can be made based on these analyses:

- If self-polarized mho elements had been protecting the line, the line would not have been cleared. Both relays would have restrained for this operation and both relays indicate that the fault is behind the Relay 2 location, not on the line.
- The Relay 2 compensator element clears the fault, but further analysis indicates that this happened because a voltage phase reversal occurred, not because the fault was within the reach of the element.
- A negative-sequence directional element at both terminals declares the fault on the line. While this disagrees with the self-polarized mho element, the $Z_{2}$ element indicates the position of the switch.
- The maximum current seen by each relay was nearly the same. This does not agree with a phase-to-phase fault at the switch location, according to the fault study provided.


## D. Relay 3 Analysis

Relay 3, also using a compensator distance element, protects a radially fed line from the same bus as Relay 2. Relay 3 undesirably tripped on Zone 1 for this cross-connect condition. The oscillography for the event from Relay 3 is shown in Fig. 14.


Fig. 14. Relay 3 Event Report Oscillography
The Zone 1 reach in service for Relay 1 at the time of the cross-connect condition was 120 percent of the line impedance
( $0.16 \angle 48^{\circ}$ ohms secondary). Also, we note that the fault detector $(50 \mathrm{~L})$ is set at 60 A primary, which is below the balanced load current of 170 A .

Relay 3 is connected to the same bus voltage transformer as Relay 2, and therefore also sees a $\left|\mathrm{V}_{2}\right|$ greater than $\left|\mathrm{V}_{1}\right|$. The $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ values are virtually the same as the measured $\left|\mathrm{V}_{1}\right|$ and $\left|\mathrm{V}_{2}\right|$ at the relay because the load current is small. Fig. 15 shows the $\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ values during the event.


Fig. 15. $\quad\left|\mathrm{V}_{1} \mathrm{C}\right|$ and $\left|\mathrm{V}_{2} \mathrm{C}\right|$ for Relay 3
Because $\left|\mathrm{V}_{2} \mathrm{C}\right|$ is greater than $\left|\mathrm{V}_{1} \mathrm{C}\right|$, the compensator element operates. Had the fault detector been set above load, the compensator element would have been prevented from operating.

Next, we turn our attention to the $Z_{2}$ element of Relay 3 in Fig. 16. Interestingly, the negative-sequence directional element declares the fault in the reverse direction. This is because the fault creates negative-sequence voltage from the depressed voltages on two of the phases feeding this load (Phases A and B). This creates negative-sequence current due to the lower phase current being drawn by the load on these two phases. The negative-sequence element declares reverse for this scenario.


Fig. 16. Sign Evaluation of $Z_{2}$ Element for Relay 3
The relay applied here is an older vintage microprocessorbased implementation of a compensator distance relay, and the compensator element is not supervised by a directional element. However, a newer version of the microprocessorbased compensator relay, available since 1999, does supervise the compensator distance element with a negative-sequence directional element. Had the newer relay been installed at the time of the event, Relay 3 would not have tripped.

Finally, we look at the self-polarized mho element operation for this fault. Fig. 17 shows the mho element direction as forward. The apparent reach using (9) evaluates to 22.186 ohms secondary, which is much greater than the Zone 1 reach setting of 0.16 ohms secondary. This condition would not have operated a self-polarized mho element.


Fig. 17. Mho Element Directional Evaluation for Relay 3
In summary, the Relay 3 misoperation could have been prevented by the following:

- Setting fault detectors above load (possible with existing relay in service).
- Supervising compensator elements with a
negative-sequence directional element (requires newer model relay).
- Using mho elements (requires newer model relay).

The analysis in this section shows us that the cross-connect fault led to unexpected results when compared with a phase-to-phase fault. Some of our questions include:

- Why was there more $\left|\mathrm{V}_{2}\right|$ present than $\left|\mathrm{V}_{1}\right|$ at Bus 2?
- Why did the compensator distance element at Relay 1 see the fault as behind Relay 2 (external to the protected line)?

(a2)

(a3)

- Why did the $\mathrm{Z}_{2}$ directional elements appear to disagree with the self-polarized mho element in terms of direction?
- Why is the fault current nearly equal between Relay 1 and Relay 2?


## IV. Symmetrical Component Analysis

From Section III, we can see that a cross-connect condition is similar to, but not necessarily the same as, a phase-to-phase fault. In this section we explore the differences and similarities between the two faults. For these examples, we assume no load conditions, no fault resistance, ABC phase rotation, and no source impedance. We neglect impedance angles for simplification. We also choose to evaluate the classical Phase-B-to-Phase-C fault rather than a Phase-A-to-Phase-B fault.

## A. Single-Source System

In this section, we start by comparing a phase-to-phase fault and a cross-connect fault for a single-source system, as shown in Fig. 18. In this system, the line impedance is 2 pu and the fault condition occurs at 50 percent of the line impedance. Fig. 18 shows a single-source circuit with a phase-to-phase fault on the left (a1 to a4) and a cross-connect condition on the right (b1 to b4). In the figure, (a1) and (b1) compare the phase connections for the two fault conditions. The phase-to-phase fault in (a1) is a shunt fault and we can see that current will flow in Phase B and Phase C. However, we can observe that no current will be allowed to flow for the cross-connect condition in (b1).

(b2)

(b4)

$$
\mathrm{I}_{1}=\mathrm{I}_{2}=0 \mathrm{pu}
$$

Fig. 18. Comparison Between Phase-to-Phase and Cross-Connect Fault for a Single Source

The sequence network connections for the two conditions are shown in (a2) and (b2) of Fig. 18. Reference [5] covers symmetrical component derivation for phase-to-phase faults. The cross-connect condition is similar in that positive- and negative-sequence symmetrical components are involved, but there is no shunt connection between the two networks as there is in the phase-to-phase fault. Instead, the cross-connect condition rolls Phase B and Phase C, which in turn creates a cross connection in the sequence networks. No current is allowed to flow in-see (b4) and (b2)-because there is not a completed circuit. However, the voltage from (b3) goes from $1 \mathrm{pu}\left|\mathrm{V}_{1}\right|$ and $0 \mathrm{pu}\left|\mathrm{V}_{2}\right|$ on the left side of the cross-connect fault to $1 \mathrm{pu}\left|\mathrm{V}_{2}\right|$ and $0 \mathrm{pu}\left|\mathrm{V}_{1}\right|$ on the right side of the crossconnect fault. This follows (b1) and (b2) because the positivesequence source on the left becomes a negative-sequence voltage on the right.

In (a3) and (a4) for the phase-to-phase fault, we see that $\left|V_{1}\right|$ and $\left|V_{2}\right|$ are equal at the fault point ( 0.5 pu ) and $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ are equal and opposite. Because $\mathrm{Z}_{1 \mathrm{~L}}$ and $\mathrm{Z}_{2 \mathrm{~L}}$ are considered equal in transmission lines, we can see that $\left|\mathrm{V}_{1}\right|$ and $\left|\mathrm{V}_{2}\right|$ will always be equal to 0.5 pu at the point of fault. We also note that it is not possible for $\left|\mathrm{V}_{2}\right|$ to be greater than $\left|\mathrm{V}_{1}\right|$.

## B. Two-Source System: Fault at Midpoint

For current to be present in a cross-connect condition, sources must be present on both sides of the cross-connect location. In this section, Fig. 19 will be used to compare a

(a2)


(b2)

phase-to-phase fault (a1 to a4) and a cross-connect fault (b1 to b4) in a two-source system with a fault at the midpoint. The line impedance is 2 pu and faults are located at 50 percent of the line impedance.

In the figure, (a1) and (b1) show the phase connections for the two fault types, while (a2) and (b2) show the sequence network connections for the two fault types. In (13) and (14), we solve for $I_{1 L}$ and $I_{1 R}$ for the phase-to-phase fault.

$$
\begin{align*}
& \mathrm{I}_{1 \mathrm{~L}}=\frac{1}{\mathrm{Z}_{1 \mathrm{~L}}+\mathrm{Z}_{2 \mathrm{~L}}}=0.5  \tag{13}\\
& \mathrm{I}_{1 \mathrm{R}}=\frac{1}{\mathrm{Z}_{1 \mathrm{R}}+\mathrm{Z}_{2 \mathrm{R}}}=0.5 \tag{14}
\end{align*}
$$

Because the fault is at the midpoint of the total impedance, the currents are equal in magnitude for this phase-to-phase fault.

Now, we solve for current in the cross-connect network by summing voltage drops in the loop circuit, starting with the left positive-sequence source and following current flow around the loop. Because this is a series connection, there is only one loop current to solve for in (15) and (16).

$$
\begin{gather*}
0=1-\mathrm{I}_{1 \mathrm{~L}}\left(\mathrm{Z}_{1 \mathrm{~L}}+\mathrm{Z}_{2 \mathrm{R}}\right)+1-\mathrm{I}_{1 \mathrm{~L}}\left(\mathrm{Z}_{1 \mathrm{R}}+\mathrm{Z}_{2 \mathrm{~L}}\right)  \tag{15}\\
\mathrm{I}_{1 \mathrm{~L}}=\frac{2}{\mathrm{Z}_{1 \mathrm{~L}}+\mathrm{Z}_{2 \mathrm{R}}+\mathrm{Z}_{1 \mathrm{R}}+\mathrm{Z}_{2 \mathrm{~L}}}=0.5 \tag{16}
\end{gather*}
$$


(b4)


Fig. 19. Comparison Between Phase-to-Phase and Cross-Connect Faults at Electrical Center for Two Sources

The current magnitude is identical for both fault types, as shown in (a4) and (b4) of Fig. 19. Therefore, the voltage drop from the source to the fault point is also the same for each fault, as shown in (a3) and (b3). The voltage drops for $\mathrm{V}_{1 \mathrm{~L}}$ in (17) and $V_{2 L}$ in (18) are calculated for both fault types.

$$
\begin{align*}
& \mathrm{V}_{1 \mathrm{~L} @ \mathrm{FAULTPOINT}}=1-\mathrm{I}_{1 \mathrm{~L}} \cdot \mathrm{Z}_{1 \mathrm{~L}}=0.5  \tag{17}\\
& \mathrm{~V}_{2 \mathrm{~L} @ \mathrm{FAULTPOINT}}=-\mathrm{I}_{2 \mathrm{~L}} \cdot \mathrm{Z}_{2 \mathrm{~L}}=0.5 \tag{18}
\end{align*}
$$

We can see from Fig. 19 that a cross-connect fault is the same as a phase-to-phase fault when the cross-connect switch is at the electrical center of the system.

## C. Two-Source System: Fault at 75 Percent of Total Impedance

Next we look at the two fault types when the fault point is not at the electrical center of the system, as shown in Fig. 20. The fault point is placed at 75 percent of the total line impedance from the left terminal. To further simplify the diagram, we remove the impedance symbols from the diagram. The line length represents the electrical distance to the fault. In Fig. 20, (a1) and (b1) represent the phase connections for the two fault types, while (a2) and (b2) represent the sequence network connections for the two fault types.

The phase-to-phase fault current magnitude is higher from the right terminal because the fault is closer to the right terminal, as shown in (a4). This also creates a steeper voltage

(a2)

(a4)

drop from the right terminal to the fault when compared with the left terminal to the fault, as shown in (a3). However, the cross-connect fault voltage profile (b3) and current profile (b4) are nearly unchanged from Fig. 19. The exception is that the switch location changes the point at which the current direction changes.

While the current magnitude, as seen in (b4), and the voltage magnitude profile, as seen in (b3), are the same in Fig. 19 as in Fig. 20, the location of the cross-connect fault is no longer at the same location as the electrical center of the system. The shaded grey area in Fig. 20 (b1 to b4) represents the zone between the electrical center of the system and the cross-connect switch. In this zone, the positive-sequence voltage drop from the left source to the cross-connect switch exceeds 0.5 pu. The negative-sequence voltage rise exceeds 0.5 pu . Relays in the grey zone see a voltage phase reversal. From the analysis in Section III, we can see that Relay 2 and Relay 3 were in the area between the electrical center of the system and the cross-connect switch position.

For relays outside of the grey zone, the cross-connect condition is no different than a phase-to-phase fault at the electrical center of the system. From the analysis in Section III, we can see that Relay 1 was outside of the grey zone because it did not experience a phase reversal. It saw the fault at the electrical center of the system, which was behind Relay 2.


Fig. 20. Comparison Between Phase-to-Phase and Cross-Connect Faults at 75 Percent Line Impedance for Two Sources

Based on these observations, we see that we can also calculate the fault current for a cross-connect condition by simply simulating a phase-to-phase fault at the center of the system. This is done by summing all impedances and dividing by 2 , as shown in (19).

$$
\begin{equation*}
\mathrm{I}_{1 \mathrm{~L}}=\frac{1}{\frac{\mathrm{Z}_{1 \mathrm{~L}}+\mathrm{Z}_{1 \mathrm{R}}+\mathrm{Z}_{2 \mathrm{~L}}+\mathrm{Z}_{2 \mathrm{R}}}{2}} \tag{19}
\end{equation*}
$$

Notice that (19) is identical to (16). We can also further reduce (16) and (19) by noting that $Z_{1}=Z_{2}$, as shown in (20).

$$
\begin{equation*}
\mathrm{I}_{1 \mathrm{~L}}=\frac{1}{\mathrm{Z}_{1 \mathrm{~L}}+\mathrm{Z}_{1 \mathrm{R}}} \tag{20}
\end{equation*}
$$

## V. Relay Performance During a Cross-Connect Fault

Next we look at how mho elements, compensator elements, and $Z_{2}$ directional elements perform during a Phase-B-to-Phase-C cross-connect fault in which the cross-connect switch is at a different location than the electrical center of the system. Fig. 21a shows an example system for this test with three separate switch positions considered. Note that Relay 2 in our sample system has a weak source behind it, and Relay 1 has a strong source behind it. This is similar to the system analyzed in Section III.

In this sample system, there is no load and the impedances are fully inductive. The total fault current for the crossconnect fault is given by (21).

$$
\begin{gather*}
\mathrm{I}_{1 \mathrm{~L}}=\frac{1}{\mathrm{Z}_{1} \text { Total }}  \tag{21}\\
\mathrm{I}_{1 \mathrm{~L}}=\frac{1}{3 \angle 90^{\circ}+1 \angle 90^{\circ}+1 \angle 90^{\circ}}=0.2 \angle-90^{\circ}
\end{gather*}
$$

How the voltage drop to Relay 2 is calculated depends on the location of the cross-connect switch. If the switch is to the left of Relay 2 (Switch Position 1), we sum voltage drops from the right source. The Relay $2 \mathrm{~V}_{1}$ is shown in (22) and the Relay $2 \mathrm{~V}_{2}$ is shown in (23) for Switch Position 1.

$$
\begin{gather*}
\text { Relay } 2 \mathrm{~V}_{1}=1-\left(0.2 \angle-90^{\circ}\right) \cdot\left(2 \angle 90^{\circ}\right)=\left(0.6 \angle 0^{\circ}\right)  \tag{22}\\
\text { Relay } 2 \mathrm{~V}_{2}=\left(0.2 \angle-90^{\circ}\right) \cdot\left(2 \angle 90^{\circ}\right)=\left(0.4 \angle 0^{\circ}\right) \tag{23}
\end{gather*}
$$

If the switch is to the right of Relay 2 (Switch Positions 2 and 3), we sum voltage drops from the left source. The Relay $2 \mathrm{~V}_{1}$ is shown in (24) and the Relay $2 \mathrm{~V}_{2}$ is shown in (25) for Switch Positions 2 and 3.

$$
\begin{gather*}
\text { Relay } 2 \mathrm{~V}_{1}=1-\left(0.2 \angle-90^{\circ}\right) \cdot\left(3 \angle 90^{\circ}\right)=\left(0.4 \angle 0^{\circ}\right)  \tag{24}\\
\text { Relay } 2 \mathrm{~V}_{2}=\left(0.2 \angle-90^{\circ}\right) \cdot\left(3 \angle 90^{\circ}\right)=\left(0.6 \angle 0^{\circ}\right) \tag{25}
\end{gather*}
$$

If the switch is located to the left of Relay 2 (Switch Position 1), current flows into the nonpolarity side of the CT, Relay $2 \mathrm{I}_{1}=0.2 \angle 90^{\circ}$, and Relay $2 \mathrm{I}_{2}=0.2 \angle-90^{\circ}$. If the switch
is located on the right side of Relay 1 (Switch Positions 2 and 3), the current flows into the polarity side of the CT, Relay 2 $\mathrm{I}_{1}=0.2 \angle-90^{\circ}$, and Relay $1 \mathrm{I}_{2}=0.2 \angle 90^{\circ}$.

Relay 1 voltage and currents can be calculated similarly based on the cross-connect switch position.

(b)


Fig. 21. Sample System With Symmetrical Components Voltage Profile
Fig. 21b shows the sample system with the symmetrical component voltage profile. The arrow at the top of the voltage profile indicates which $\left|\mathrm{V}_{1}\right|$ voltage will be used at the relay location based on the switch position. For example, Relay 1 $\left|\mathrm{V}_{1}\right|$ can either be equal to $\mathrm{V}_{1 \mathrm{R}}\left(0.8 \angle 0^{\circ}\right)$ if the cross-connect switch is to the left of Relay 1 (Switch Position 1 or 2) or $\mathrm{V}_{1 \mathrm{~L}}$ $\left(0.2 \angle 0^{\circ}\right)$ if the switch is to the right of Relay 1 (Switch Position 3).

From Fig. 21 b and taking into account the proper $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ current phase angle based on the switch position, we can graphically show the phasors that each relay will see and evaluate protection element performance. For this sample system, we set the reach of the compensator and mho distance element to 100 percent of the line $\left(1 \angle 90^{\circ}\right)$. Fig. 22 shows the phasors seen by each relay based on switch position. The compensated phasors for the negative-sequence directional element, the mho distance element, and the compensator distance element have also been included. In addition, we include the $V_{B C}$ memory voltage phasor to evaluate a positive-sequence memory-polarized element.


Fig. 22. Relay 1 and Relay 2 Phasors for Cross-Connect Fault

The following observations can be made from Fig. 22:

- The negative-sequence directional element correctly identifies the location of the cross-connect switch. For a cross-connect switch in Position 1, Relay 2 sees the fault as reverse because $\mathrm{I}_{2} \mathrm{C}$ is in phase with $\mathrm{V}_{2}$. When the cross-connect switch is in Position 2 or 3, Relay 2 sees the fault as forward because $\mathrm{I}_{2} \mathrm{C}$ and $\mathrm{V}_{2}$ are $180^{\circ}$ out-of-phase. Relay 1 also correctly identifies the direction of the switch.
- The self-polarized mho element for Relay 2 sees the fault at -0.5 ohms regardless of where the switch is located ( 1,2 , or 3 ). The relay evaluates negative because $\mathrm{I}_{\mathrm{BC}} \mathrm{C}$ and $\mathrm{V}_{\mathrm{BC}}$ are always 180 degrees out-of-
phase. The self-polarized mho element for Relay 1 sees the fault at 1.5 ohms, which is outside the Zone 1 reach of the element, regardless of where the switch is located. The relay evaluates positive because $\mathrm{I}_{\mathrm{BC}} \mathrm{C}$ and $V_{B C}$ are always in phase. Neither self-polarized mho element for Relay 1 or Relay 2 operates for a cross-connect fault in this sample system.
- The compensator elements operate when a phase reversal is experienced by a relay. When the physical location of the compensator relay is between the electrical center and the cross-connect switch position, the compensator element operates regardless of the Zone 1 reach setting.
- A line current differential relay will isolate the line section affected by a cross-connect condition.
While the self-polarized mho element has been evaluated up to this point due to its relative ease of calculation, the positive-sequence memory-polarized mho element, which is typically implemented in modern-day digital relays, has not. The equation for a memory-polarized element is given in (26). We choose to look at the MBC element because it follows our sample system.

$$
\begin{align*}
\operatorname{MBC}_{(\text {mem })} & =\frac{\operatorname{Re}\left(\mathrm{V}_{\mathrm{BC}} \cdot \overline{\mathrm{~V}_{\mathrm{BC}(\mathrm{mem})}}\right)}{\operatorname{Re}\left(1 \angle \mathrm{MTA} \cdot \mathrm{I}_{\mathrm{BC}} \cdot \overline{\mathrm{~V}_{\mathrm{BC}(\mathrm{mem})}}\right)}  \tag{26}\\
\mathrm{V}_{\mathrm{BC}(\mathrm{mem})} & =-\mathrm{j} \sqrt{3} \cdot \mathrm{~V}_{1 \mathrm{~A}(\mathrm{mem})}
\end{align*}
$$

Assuming a balanced three-phase voltage prior to the fault, $\mathrm{V}_{\mathrm{BC}(\mathrm{mem})}=\sqrt{3} \angle-90^{\circ}$. Looking at Relay 2 for Switch Position 1, the numerator evaluates to (27).

$$
\begin{equation*}
\operatorname{Re}\left(\left(0.346 \angle-90^{\circ}\right) \cdot\left(\sqrt{3} \angle 90^{\circ}\right)\right)=0.6 \tag{27}
\end{equation*}
$$

The denominator evaluates to (28).

$$
\begin{equation*}
\operatorname{Re}\left(\left(1 \angle 90^{\circ}\right) \cdot\left(0.693 \angle 0^{\circ}\right) \cdot\left(\sqrt{3} \angle 90^{\circ}\right)\right)=-1.2 \tag{28}
\end{equation*}
$$

The final result of the memory-polarized mho element for Relay 1 at Switch Position 1 is achieved with (29).

$$
\begin{equation*}
\frac{0.6}{-1.2}=-0.5 \tag{29}
\end{equation*}
$$

If we move the switch to point at Positions 2 or 3, the $\mathrm{V}_{\mathrm{BC}}$ and $\mathrm{I}_{\mathrm{BC}}$ phasors shift by 180 degrees. Looking at Relay 2 for Switch Positions 2 and 3, the numerator evaluates to (30).

$$
\begin{equation*}
\operatorname{Re}\left(\left(0.346 \angle 90^{\circ}\right) \cdot\left(\sqrt{3} \angle 90^{\circ}\right)\right)=-0.6 \tag{30}
\end{equation*}
$$

The denominator evaluates to (31).

$$
\begin{equation*}
\operatorname{Re}\left(\left(1 \angle 90^{\circ}\right) \cdot\left(0.693 \angle 180^{\circ}\right) \cdot\left(\sqrt{3} \angle 90^{\circ}\right)\right)=1.2 \tag{31}
\end{equation*}
$$

The final result of the positive-sequence memory-polarized mho element at Switch Positions 2 and 3 is given in (32).

$$
\begin{equation*}
\frac{-0.6}{1.2}=-0.5 \tag{32}
\end{equation*}
$$

The memory-polarized mho element operates identically to the self-polarized mho element in terms of overall evaluation ( -0.5 ohms). However, the denominator term changes sign depending on the location of the cross-connect switch. Because the denominator is positive for Switch Positions 2 and $3\left(\mathrm{~V}_{\mathrm{BC}(\mathrm{mem})}\right.$ is in phase with $\left.\mathrm{I}_{\mathrm{BC}} \mathrm{C}\right)$ and the calculated reach ( -0.5 ohms) is less than the set point ( 1 ohm ), the positive-sequence memory-polarized mho element operates at Relay 2. Because the denominator is negative at Switch Position 1 ( $\mathrm{V}_{\mathrm{BC}(\mathrm{mem})}$ and $\mathrm{I}_{\mathrm{BC}} \mathrm{C}$ are 180 degrees apart), the element restrains.

At Relay 1, the calculated positive-sequence memory-polarized reach is always 1.5 ohms, so the calculated reach is never lower that the set point of 1 ohm . Therefore,
even though the denominator is positive for Switch Positions 1 and 2 , the element is prevented from operating.

The positive-sequence memory-polarized element will operate, regardless of the reach setting, if the electrical center is behind the relay and the switch position is in front of the relay. It may operate if the electrical center and the crossconnect switch are in front of the relay and the electrical center is within the reach setting. The element will not operate if the cross-connect switch position is behind the relay.

While the math is easier for a self-polarized mho element, it operates differently from a memory-polarized element for a cross-connect condition. To accurately simulate a relay's response, the correct polarizing quantities should be used, especially for nontraditional faults such as a cross-connect condition. However, there is still value in simulating and comparing both the self- and memory-polarized mho elements. If the two elements respond in a drastically different manner (as in this case), it is an indication that the voltage changed unexpectedly during the fault. Reference [6] shows another example in which comparing self- to memory-polarized mho elements helped identify a nontraditional fault type.

Fig. 23 shows a summary of relay element operation for the sample system in this paper. The light shaded area shows where an individual relay operates for a cross-connect condition. The dark shaded area shows where both relays will operate.

If we compare the results for a switch at Position 2 in our sample system with the results from the event analysis in Section III, we see similar results. This answers our questions from the end of Section III, as follows:

- The Relay 2 compensator operates for a phase reversal. The phase reversal occurs because the electrical center of the system is behind Relay 2 and the cross-connect switch is in front of Relay 2. The Relay 1 compensator restrains because the apparent fault is outside its set reach.
- The Relay 1 self-polarized mho element and the Relay 2 self-polarized mho element each see the fault behind Relay 2. This causes the Relay 1 self-polarized mho element to underreach the switch position and the Relay 2 self-polarized mho element to see the fault in the reverse direction.
- The Relay 1 and Relay 2 negative-sequence directional declare forward and point to the crossconnect switch. Although $\left|\mathrm{V}_{2}\right|$ is affected by the cross-connect condition, the phase angle relationship between $\mathrm{V}_{2}$ and $\mathrm{I}_{2} \mathrm{C}$ is unaffected.
- The fault current seen by Relay 1 and Relay 2 is the same magnitude. The cross-connect fault is a series circuit between the two sources, so the current is equal at each relay location.
The model explains the operation and lack of operation for all protection elements evaluated in Section III.


Fig. 23. Summary of Relay 1 and Relay 2 Operation for the Sample System

## VI. CONCLUSION

Cross-connect faults, while similar to phase-to-phase faults, differ in that the negative-sequence voltage measured by the relay can exceed the positive-sequence voltage. This causes a phase voltage reversal and occurs when the relay is located between the electrical center of the system and the location of the cross-connect fault. These faults, while rare, do present a challenge for phase distance relays. The following is a summary of how commonly applied protection elements respond:

- Compensator distance elements operate for a voltage phase reversal, regardless of the reach setting.
- Positive-sequence memory-polarized phase-pair mho elements operate for a voltage phase reversal, regardless of reach setting, if the electrical center is behind the relay and the cross-connect fault is located in front of the relay.
- Positive-sequence memory-polarized phase-pair mho elements may operate if the electrical center and cross-connect fault location is in front of the relay and the electrical center is within the reach setting of the relay.
- Positive-sequence memory-polarized phase-pair mho elements do not operate if the cross-connect fault is located behind the relay.
- Negative-sequence impedance-based directional elements always declare the proper physical location of the cross-connect fault point (forward or reverse).
- Line current differential elements properly operate for a cross-connect fault occurring on a protected line.

A line protection scheme of compensator distance elements is dependable for a cross-connect fault, but it is not secure. Security can be improved by applying appropriately set fault detectors and supervising the compensator distance elements with negative-sequence impedance-based directional elements. Security is severely compromised when the relay is located between the electrical center and the cross-connect fault location. Fault detectors must be appropriately set for this condition, although this may not be feasible in all situations. The worst case is when several compensator distance relays protect various lines located between the electrical center and the cross-connect fault location. The potential for tripping out several lines exists.

The electrical center can be determined by reducing a network system to a two-source equivalent of a line. Short-circuit programs can provide an equivalent wye, and the two-source equivalent can then be obtained from a wye-delta transformation of the impedances. See [7] for more details.

Line current differential schemes are secure and dependable for cross-connect faults. However, because backup distance elements are often applied, the security issues for cross-connect faults remain.

Good engineering judgment is required when applying distance elements to protect lines and transformers to limit exposure to problems caused by cross-connect faults. Considerations include identifying important customer loads that should not be interrupted and identifying the potential for larger system-wide outages. Compromises must be made, as this is part of the art of protective relaying. Finally, proper operating procedures must be followed to limit exposure to cross-connect faults.

## VII. APPENDIX

The improved performance of the compensator distance element over the phase-pair mho element when protecting a line through a transformer merits some additional discussion. The compensator distance element reaches through a deltawye transformer bank for phase-to-phase, phase-to-phase-to-ground, and three-phase faults. The required reach setting can be determined by adding the transformer impedance directly to the line impedance. The compensator distance element measures the impedance through the transformer for all phase faults and does not overreach on phase-to-ground faults.

Equations (4) and (5) can help us understand why the phase-to-phase element is not affected by a delta-wye transformer. First, consider either a delta-delta or a wye-wye transformer. The condition for the phase-to-phase element operating is determined using (7).

$$
\begin{equation*}
\frac{\left|\mathrm{V}_{2 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{2 \mathrm{~A}}\right|}{\left|\mathrm{V}_{1 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{1 \mathrm{~A}}\right|}>1 \tag{33}
\end{equation*}
$$

Next consider a delta-wye transformer with ANSI standard connections, where the high-voltage quantities lead the low-voltage quantities by 30 degrees. Assume a fault on the high-voltage side with the CTs and VTs for the relay located on the low-voltage side. Using symmetrical components to calculate the sequence voltages and currents seen by the relay results in (34) and (35).

$$
\begin{gather*}
\mathrm{V}_{1} \mathrm{C}=\left(\mathrm{V}_{1 \mathrm{~A}} \cdot 1 \angle-30^{\circ}\right)-\mathrm{Z}_{\mathrm{IR}} \cdot\left(\mathrm{I}_{1 \mathrm{~A}} \cdot 1 \angle-30^{\circ}\right)  \tag{34}\\
\mathrm{V}_{2} \mathrm{C}=\left(\mathrm{V}_{2 \mathrm{~A}} \cdot 1 \angle 30^{\circ}\right)-\mathrm{Z}_{\mathrm{IR}} \cdot\left(\mathrm{I}_{2 \mathrm{~A}} \cdot 1 \angle 30^{\circ}\right) \tag{35}
\end{gather*}
$$

Note that the positive-sequence current and voltage are shifted by -30 degrees, because the relay is located on the low-voltage side of the transformer. Likewise, the negativesequence current and voltage are shifted by +30 degrees. Equations (34) and (35) can be rewritten as follows:

$$
\begin{align*}
& \mathrm{V}_{1} \mathrm{C}=1 \angle-30^{\circ} \cdot\left(\mathrm{V}_{1 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{1 \mathrm{~A}}\right)  \tag{36}\\
& \mathrm{V}_{2} \mathrm{C}=1 \angle 30^{\circ} \cdot\left(\mathrm{V}_{2 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{2 \mathrm{~A}}\right) \tag{37}
\end{align*}
$$

The condition for the phase-to-phase element operating is determined using (7).

$$
\begin{equation*}
\frac{\left|\mathrm{V}_{2 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{2 \mathrm{~A}}\right|}{\left|\mathrm{V}_{1 \mathrm{~A}}-\mathrm{Z}_{1 \mathrm{R}} \cdot \mathrm{I}_{1 \mathrm{~A}}\right|} \cdot \frac{1 \angle 30^{\circ}}{1 \angle-30^{\circ}}>1 \tag{38}
\end{equation*}
$$

From (38), we can see that the magnitude of the value on the left side of the equation is identical to the magnitude on the left side of (33). The $\pm 30$ degree phase shift required in the symmetrical component analysis with a delta-wye transformer has no effect on the operating characteristic of the phase-tophase compensator distance element. The reach through the transformer can be set the same for all transformer connection types. This also holds true for the three-phase element in (8). As noted in [3], voltage-polarized mho elements properly reach through all transformer connections for three-phase faults.

## VIII. REFERENCES

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## IX. Biographies

Jon Larson is an application engineer in protection with Schweitzer Engineering Laboratories, Inc. (SEL) and is located in Marshall, Michigan. Jon received B.S.E.E. and M.S.E.E. degrees from Michigan Technological University and an M.B.A. from Eastern Michigan University. Jon worked 15 years at a major Midwest utility with responsibilities primarily in protective relaying. He has been employed with SEL since October 2002. Jon is a member of IEEE and a registered professional engineer in the state of Michigan.

Ryan McDaniel earned his B.S. in Computer Engineering from Ohio Northern University in 2002. In 1999, Ryan was hired by American Electric Power (AEP) as a relay technician, where he commissioned protective systems. In 2002, Ryan began working in the Station Projects Engineering group as a protection and control engineer. His responsibilities in this position included protection and control design for substation, distribution, and transmission equipment as well as coordination studies for the AEP system. In 2005, Ryan joined Schweitzer Engineering Laboratories, Inc. and is currently a field application engineer. His responsibilities include providing application support and technical training for protective relay users. Ryan is a registered professional engineer in the state of Illinois and a member of IEEE.

