A New Method for Fast Frequency Measurement for Protection Applications

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Abstract

This paper presents a novel approach to frequency estimation for power system applications: frequency tracking, frequency protection, and synchrophasors. We based our algorithm on autocorrelation of the waveform or correlation of the waveform and its derivative using a freely chosen window length. Because of the averaging inherent in correlation, the method has exceptional immunity to noise. We derive, explain, and illustrate the new method with simulation results. We review advantages and attributes of the new method compared with two commonly used approaches to frequency measurement: timing between the waveform zero crossings and measuring the rate of rotation of a phasor.

1 Introduction

Frequency is a key power system measurement used for frequency control and frequency protection, including load shedding and overexcitation protection. Frequency is also used in synchrophasor measurements and in protective relays and meters for frequency tracking in order to keep ac measurements, especially phasors, accurate.

One understands and unambiguously defines frequency only for a truly periodic signal. Signal x has a period T_1 if for any point in time t, the following is true:

$$x(t - T_1) \equiv x(t) \tag{1}$$

In other words, the lowest value of T_1 satisfying (1) is the fundamental frequency period of the signal x.

By definition, periodic signals are stationary, occurring only in steady states. Therefore, frequency is a steady-state term. Power systems, however, are in a state of constant fluctuation as generators oscillate around their equilibrium points in a stable power system. Major system events can cause pronounced oscillations as generators speed up and slow down with respect to the rest of the power system. Some network configurations may lead to resonant conditions causing subsynchronous oscillations. As a result, power system voltages and currents are never in a perfect steady state and, therefore, are not precisely periodic.

How do we define frequency for such practical conditions? Reference [1] is an in-depth study of the subject of dynamic frequency, but its findings are not widely applied and the paper does not relate specifically to power system frequency.

In electric power systems, a generator rotor spins at a certain rate. The rate-of-change of the rotor angular position (angular velocity) is an excellent proxy for a generator's frequency. Because of the mechanical inertia, the angular velocity is a very clean signal with a very high signal-to-noise ratio (assuming an adequate shaft speed or position sensor is applied). However, once we shift away from the mechanical definition and start looking at voltages and currents, this link to physical interpretation of generator frequency becomes weaker, especially as we move a few buses away from any given generator. Moreover, if we factor in torsional oscillations, even the mechanical interpretation of power system frequency becomes less obvious. A long generator and turbine shaft oscillating in a torsional fashion will have slightly different rates of rotation at different points along the shaft.

Switching events complicate our understanding of frequency in power systems even more. During switching events, currents and voltages are not periodic. Strictly speaking, we cannot measure the period or frequency at all during switching events.

Practical frequency measurement methods used today in power systems assume slow frequency changes compared with the rated system frequency, i.e., they assume near-stationary signals. These methods incorporate the means to "ride through" switching events using a number of heuristic approaches. The industry does not have any common definition of dynamic frequency, despite some standardization efforts [2]. Therefore, each measurement method becomes its own definition of frequency. In the absence of a dynamic frequency definition, the industry relies on tests. Common dynamic tests include frequency ramps and oscillations. Below is a brief summary of the two commonly used methods for frequency measurement and the IEEE C37.118 approach to frequency.

1.1 Frequency measurement based on zero crossings

This method times the interval between zero crossings of a waveform (Fig. 1a). Some implementations time full cycles looking at the zero crossings in the same direction (negative-to-positive and positive-to-negative), while some methods time half cycles between zero crossings in any direction. Using information from only the zero-crossing area of the waveform makes this method immune to harmonics as long as the waveform is truly periodic and does not exhibit extra zero crossings due to heavy distortions (see Fig. 1a). Often, low-pass prefiltering improves accuracy of zero-crossing detection.

This filtering introduces side effects as the filter responds to magnitude changes and causes the zero crossings to shift slightly as the magnitude changes. Interpolation often provides a more precise zero-crossing time estimation, especially if the sampling rate is relatively low.

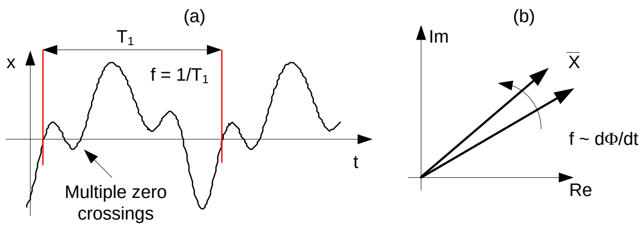

Fig. 1. Frequency measurement methods: zero crossings (a) and rate-of-change of phasor rotation (b).

Switching events cause the zero crossings to shift instantly, creating very large frequency measurement errors when using this method. These errors must be filtered out by rejecting wrong measurements rather than by averaging, hence the use of various heuristic approaches to validating or rejecting such raw measurements.

Many applications average validated raw frequency measurements to improve accuracy of the final reported frequency. Averaging filters receive frozen last valid measurements if the method rejects the raw measurement for the present time.

This method, does not run the key calculations, such as raw frequency or averaged frequency, at a constant rate. It runs the calculations only when it detects a zero crossing. These event-driven calculations constitute variable sampling for the raw frequency stream of data, even with some data points temporarily missing when the validation checks mark a corresponding zero crossing as invalid for a nonperiodic waveform.

All these factors make the zero-crossing methods very convoluted. They are widely used, but are proprietary, and exhibit a strong nonlinear behavior. As a result, various implementations of this method may yield different results for any given dynamic system event.

1.2 Frequency measurement based on phasor rate of rotation

This method mimics the mechanical analogy of generator frequency by measuring an input phasor and calculating the rate of angular rotation of the phasor (Fig. 1b). This rate is proportional to frequency.

One can run this method at a fixed rate, such as each time a new input waveform sample becomes available. This makes any raw frequency postfiltering more straightforward. As with any frequency measuring method, the method behaves poorly for nonperiodic waveforms. During switching events, the phasor angle shifts abruptly. This phase shift causes very large errors and calls for validation/rejection logic for the raw frequency. Alternatively, one may fit a straight line to a series of angle positions and report frequency as the slope of such a line. In yet another approach, one may fit a second order function and report frequency and rate-of-change of frequency from the parameters of such a best-fit function.

The major problem with this method is that measuring the phasor correctly and properly rejecting harmonics requires knowing the waveform frequency. We can resolve this feedback loop (to measure frequency from the phasor we need to know the frequency) by using iterations or by quickly adjusting the frequency tracking. This method is used in practice, but it does not show any significant improvements over the zero-crossing method.

1.3 C37.118 approach to frequency

The IEEE C37.118.1-2011 synchrophasor standard attempted to define frequency by including a reference measurement method (informative annex) and specifying performance requirements for the measurement (normative part). This attempt to define frequency indirectly via test conditions proved difficult as shown by the fact that the 2011 requirements were relaxed with an amendment issued in 2014 [2]. While the original requirements were impossible to meet, the newly relaxed requirements open the possibility of compliant implementations yielding different frequency measurements for the same test conditions.

2 Desired attributes for a frequency measurement method

A good frequency measurement method:

- Meets application requirements for accuracy and response time under realistic system conditions. Accuracy in the order of a few mHz or better may be required. Typical system conditions include frequency excursions of ±5 Hz and frequency ramps of up to ±15 Hz/s. Inertia-free inverter-based power sources pose higher requirements than the traditional requirements listed above.
- Can be calculated at will and tied to internal device processes such as the waveform sampling process, rather than in response to an external event (e.g., zero crossing).
- Does not require any prior knowledge (approximation) of frequency and does not need to use an iterative process to obtain the accurate value of frequency.
- Quickly recovers from switching events.
- Incorporates a quality-of-measurement index signifying if the input waveform is periodic. This index allows invalid measurements to be easily identified.
- Allows straightforward balance between accuracy and speed of response, e.g., by adjusting a low-pass filter included in the method.
- Does not require any waveform prefiltering and thus avoids any artifacts of such prefiltering.
- Is accurate for periodic signals, even if heavily distorted by harmonics, including cases of multiple zero crossings within each fundamental frequency period.
- Allows natural usage of all three phase signals.

This paper presents a new frequency measurement method that meets the above requirements.

3 Defining power system frequency

Because frequency is a reciprocal of period, we refer to frequency measurement and period measurement interchangeably to describe the same measurement task.

3.1 Common sense approach to power system frequency

A period of a repeating waveform is the time shift that needs to be applied to the waveform in order for it to "become itself" (see (1)). In other words, the period is a shift that maximizes signal autocorrelation. This definition works as long as the signal is periodic, even if it includes harmonics and/or a dc component.

In practice, we need to decide on the length of the autocorrelation window. The exact waveform period is the logical choice. We cannot use half a period because the input waveform in general does not have to be antisymmetrical, i.e., x(t–T/2) does not have to equal –x(t). Using multiple periods adds delay and theoretically does not provide any extra data.

3.2 Formalized approach to power system frequency

A period of a repeating waveform is the time shift T_1 for which an integral over a time period T_1 of the product of the waveform and the waveform delayed by T_1 is at its maximum. In other words:

$$A(t,T) = \int_{t-T}^{t} x(t) \cdot x(t-T)dt \tag{2a}$$

$$T_1(t) = T \text{ for which } A(t,T) \text{ is at its maximum} \tag{2b}$$

However, as we shift the waveform by T and integrate over T in (2a), we increase the integral when we increase T, not because the correlation is better, but because we integrate over a longer time interval. Therefore, it is imperative to normalize the integral as follows.

3.3 Proposed definition of power system frequency

A period of a repeating waveform is the time shift T_1 for which an integral over a time period T_1 of the product of the waveform and the waveform delayed by T_1, normalized with the wave magnitude, is at its maximum. In other words:

$$A(t,T) = \frac{\int_{t-T}^{t} x(t) \cdot x(t-T)dt}{0.5\int_{t-2T}^{t} x^2(t)dt} \tag{3}$$

A truly periodic waveform $A(t,T_1)$ per (3) is exactly 1. For a near-periodic waveform, such as during a frequency ramp and/or a magnitude ramp or oscillation, $A(t,T_1)$ per (3) is below but close to 1. For a nonperiodic waveform, such as for a phase or magnitude jump, $A(t,T_1)$ per (3) is considerably below 1. Therefore, the value of $A(t,T_1)$ is a good measurement quality indicator. We may consider a waveform periodic, and thus mark the measured period T_1 as valid, if $A(t,T_1) > 0.95$ for example. The exact threshold depends on the frequency and magnitude rates of change we want to follow.

We can run method (3) at any arbitrary point in time, t, which is convenient for synchrophasor applications. It returns the period and its validity, i.e., the information, if the waveform is actually periodic at the time t.

The method uses a sliding data window of two actual periods. We use this data window length as it takes not less than two full periods to verify if the waveform is truly periodic.

Method (3) is well suited for off-line applications, but it may be computationally too demanding in real time because of the need to find the function maximum in (3). The next section proposes yet another method that yields equally good results but is numerically simpler. This alternative method searches for the zero of a function rather than the maximum.

3.4 Definition of power system frequency suitable for efficient implementation

A periodic signal contains the fundamental frequency component and a number (potentially infinite) of harmonics (signals of frequencies being integer multiples of the fundamental). Each harmonic is a sinewave. Note that for a sinewave signal, the signal and its time derivative are shifted by 90 electrical degrees at the frequency of the said signal. Due to this relationship, the integral of the product of the signal and its derivative taken over the period of the said signal is zero (see the Appendix for the mathematical proof).

The above observation allows us to define the period as follows. A period of a repeating waveform T_1 is the length of the integration window for which the integral of the product of the signal (x) delayed by T_1 and its time derivative (x') is zero. In other words:

$$B(t,T) = \int_{t-T}^{t} x'(t) \cdot x(t-T)dt \tag{4a}$$

$$T_1(t) = T \text{ for which } B(t,T) \text{ is at zero} \tag{4b}$$

We can apply the delay to the signal derivative instead of the signal and obtain an equally good method as follows:

$$B(t,T) = \int_{t-T}^{t} x(t) \cdot x'(t-T)dt \tag{4c}$$

Methods (3) and (4) are similar. Method (4), however, is numerically simpler because it requires finding a zero of the B-function (4a) instead of finding the maximum of the A-function (3).

Method (3) provides a quality-of-measurement index where the $A(t,T_1)$ value signifies if the signal is truly periodic. In that sense, (4) can be applied to find T_1 and (3) can be applied to check if T_1 is valid.

4 New power system frequency algorithm

The B-function (4a) has a peculiar property: when calculated for a period T, shorter than the actual period T_1, the function is positive, and when calculated for a period T, longer than the actual period T_1, the function is negative (see the Appendix for the mathematical proof). This observation holds true for practical frequency measurement ranges when T and T_1 do not differ much. This key observation allows a very simple implementation: if the B-function is positive, the present period estimate shall be increased, and when the B-function is negative, the present period estimate shall be decreased.

Further, the B-function does not change much when calculated for two consecutive samples (see the Appendix for the analytical expression of the B-function). As a result, instead of performing iterations for a given data set at a given point in time, we iterate using new data sets. In other words, we calculate the B-function once for any given point in time and decide to increase or decrease the period estimate for the calculation of the B-function for the next data set. If the sampling frequency is sufficiently high (such as 4 kHz or higher), the process is stable even for extremely high rates-of-change of frequency.

Ideally, the value of the period (i.e., the shift in the signal) in the new method should not be limited to integer multiples of samples. For example, if the signal frequency is 60 Hz and the sampling frequency is 10 kHz, the period is 166.6(6) samples. Calculating the B-function for $T = 166$ samples gives a positive value, e.g., 100 (positive value means 166 is below the actual period). Following our algorithm, if we increase the period estimate and calculate the B-function for $T = 167$ samples, we obtain a negative value, e.g., –49.25 (negative value means 167 is above the actual period). A simple algorithm limited to the period estimate in integer sample counts would oscillate the estimated period between 166 and 167 samples. Note that the straight average between 166 and 167 (166.5) does not reflect the true period of 166.6(6). Resampling would allow us to apply fractional shifts, but resampling is computationally expensive. To solve this, we use periods expressed in integer sample counts and apply interpolation if the B-function changes sign between two consecutive samples. In this example, we interpolate between the two points (166 samples, 100) and (167 samples, –49.25) and obtain the period of 166.67 samples. This way the algorithm constantly tracks the zero of the B-function by probing it to the left and right of the actual zero. The interpolation allows for better accuracy. If the B-function does not change signs between two consecutive samples, the algorithm uses the newest value of the period.

Finally, to increase accuracy, we apply a low-pass filter (LPF) to the period values obtained from interpolation. The LPF can be set at about 15 Hz in order to allow frequency changes as fast at 15 Hz/s. The lower the cut-off frequency, the better the accuracy and the slower the response time of the algorithm (the lag between the true and measured frequency). Low-pass filtering is a simple and effective way to control accuracy-versus-speed performance requirements. In particular, two filters can be used: one for metering purposes and the other for frequency protection or frequency tracking purposes. Fig. 2 presents the algorithm implementation.

The presented algorithm requires relatively high sampling rates, in the order of 4 to 10 kHz. We can implement it, however, with lower sampling rates using up-sampling.

The LPF in Fig. 2 can be of a finite impulse response, i.e., have a constant group delay, for which we can easily compensate. This makes the new algorithm a very good candidate for frequency measurement in synchrophasor or other time-tagged applications.

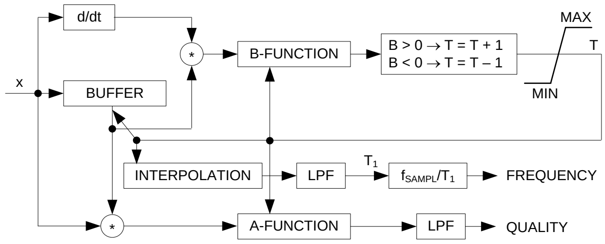

Fig. 2. Block diagram of the new frequency algorithm (T and T_1 are in samples of sampling frequency f_{SAMPL}).

5 Performance and sample results

In order to illustrate performance of this new algorithm, consider a heavily distorted waveform with frequency and magnitude changing over time (Fig. 3). In this example, the frequency declines at 20 Hz/s, exhibits a step change by 5 Hz, and later increases at 2.5 Hz/s.

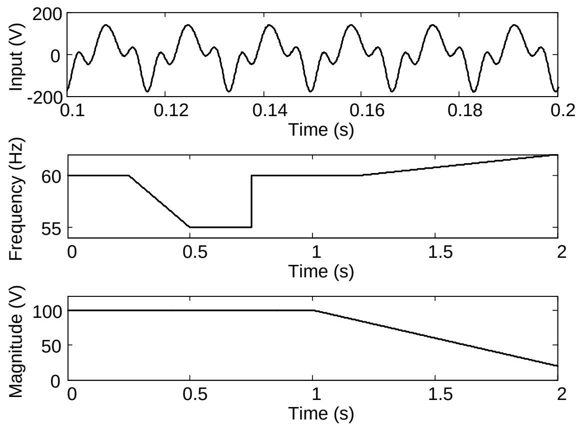

Fig. 3. Input signal used for testing: waveform (top), frequency (middle), and magnitude of fundamental frequency component (bottom).

Fig. 4a shows the actual period, the raw period in integer samples, the raw period interpolated between samples, and the final period estimate after low-pass filtering, for a fraction of the test time. Fig. 4a shows how the raw period tracks the actual period. Fig. 4b presents the actual and measured frequencies. The measurement error in the steady state is very small, on the order of 0.6 mHz.

Fig. 5 shows the actual and measured frequencies and illustrates how well the new algorithm tracks frequency despite magnitude changes and very significant harmonic distortions in the input signal.

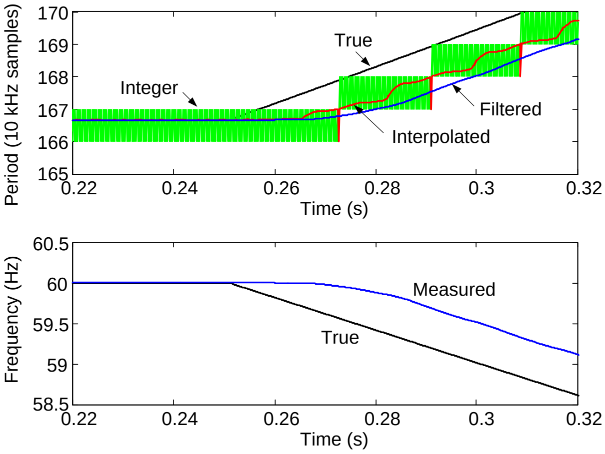

Fig. 4. New algorithm operation illustration: period (top) and frequency (bottom) for a –20 Hz/s ramp.

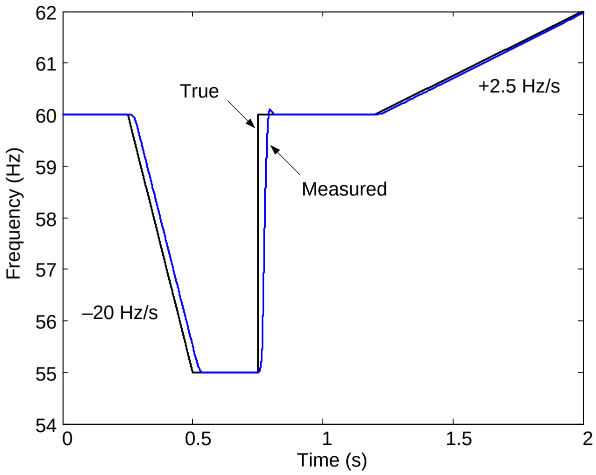

Fig. 5. Response of the new algorithm to the test of Fig. 3.

Fig. 6 illustrates frequency measurement for a switching event (phase jump by 90 degrees at $t = 0.3$ s).

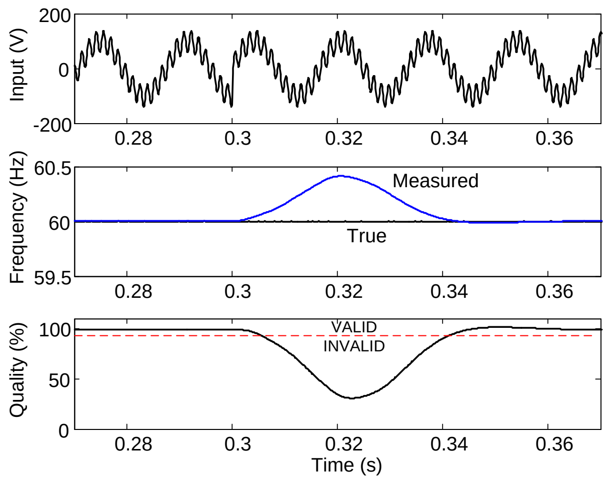

Fig. 6. Response of the new algorithm to a switching event: input waveform (top), frequency (middle), and quality-of-measurement index (bottom).

As expected for any frequency measurement method, the frequency measurement is affected during the phase jump. The new method, however, calculated a low quality-of-measurement index during the interval for which the measurement was not accurate. This low index value may engage a selected fall-back logic, such as freezing the last valid measurement until the quality-of-measurement index returns to an acceptable value. In this case, the measurement is marked as invalid for about 40 ms (time needed to flush the phase jump from the algorithm's memory).

Fig. 7 presents a sample result for an EMTP-simulated system. The network includes series compensation and a low-inertia generator. The former caused subsynchronous oscillations and the latter caused a fast-frequency ramp after the generator became islanded after clearing a fault. The new method performs very well compared with the zero-crossing approach (modeled with no prefiltering). The quality-of-measurement index correctly identifies frequency measurements that are less accurate.

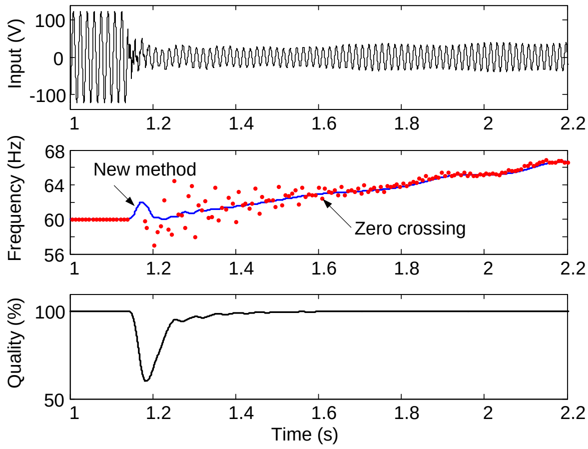

Fig. 7. Transient simulation example including subsynchronous oscillations and a low-inertia generator.

6 Conclusions

This paper proposes a definition of dynamic frequency for power system applications and derives a simple and robust implementation of frequency measurement for off-line or real-time applications. The new frequency measurement algorithm has numerous advantages and features differentiating it from other commonly used methods.

The new method is accurate and fast. It allows for a convenient ride-though for switching events by providing a numerical quality-of-measurement index. The new method has constant and low latency, making it a good candidate for synchrophasor or other time-tagged applications.

References

[1] B. Boashash, "Estimating and Interpreting the Instantaneous Frequency of a Signal," Proceedings of the IEEE, Vol. 80, No. 4, April 1992. Part 1: Fundamentals (pp. 520–538) and Part 2: Algorithms and Applications (pp. 540–568).

[2] IEEE standard C37.118.1a-2014: IEEE Standard for Synchrophasor Measurements for Power Systems.

Appendix

The presented period estimation method relies on the key observation that the B-function (4a) is negative if calculated for a period longer than the actual period, and it is positive if calculated for a period shorter than the actual period. This appendix provides a mathematical proof of this key idea.

Assume a period waveform in a steady state composed of a number of harmonics. We first look at a single harmonic. Our input signal is therefore a single frequency component as follows:

$$x(t) = A_h sin\left(\frac{2\pi h}{T_1}t + \alpha\right) \quad \text{(A-1)}$$

where:

- h is the harmonic order (h = 1, 2, 3, …).
- A_h is the peak magnitude.
- α is an arbitrary initial phase angle.
- T_1 is the true period (fundamental frequency).

The input derivative of (A-1) is as follows:

$$x'(t) = \frac{2\pi h}{T_1} A_h cos\left(\frac{2\pi h}{T_1}t + \alpha\right) \quad \text{(A-2)}$$

Substituting (A-1) and (A-2) into (4b) we obtain:

$$B(t,T) = \frac{2\pi h}{T_1}(A_h)^2 \left[\int_{t-T}^{t} sin\left(\frac{2\pi h}{T_1}(2t - T) + 2\alpha\right)dt - sin\left(\frac{2\pi h}{T_1}T\right)\right] \quad \text{(A-3)}$$

Solving for the integral we obtain:

$$B(t,T) = -\frac{\pi h}{T_1}(A_h)^2 \cdot \left[sin\left(\frac{2\pi h}{T_1}T\right) + \frac{T_1}{4\pi h}cos\left(\frac{2\pi h}{T_1}(2t - T) + 2\alpha\right)\right]_{t-T}^{t} \quad \text{(A-4)}$$

Finally, inserting the limits of integration we obtain:

$$B(t,T) = \frac{\pi h}{T_1}(A_h)^2 sin\left(\frac{2\pi h}{T_1}T\right) \cdot \left[-T + \frac{T_1}{\pi h}sin\left(\frac{4\pi h}{T_1}(t - T) + 2\alpha\right)\right] \quad \text{(A-5)}$$

Next, we analyze the sign of (A-5) depending on the ratio of T and T_1, i.e., depending on whether the estimated period (T) is shorter or longer than the actual period of the waveform (T_1).

Note that the expression (A-6),

$$sin\left(\frac{4\pi h}{T_1}(t - T) + 2\alpha\right) \quad \text{(A-6)}$$

is a variable between –1 and +1 depending on the initial angle α and a specific point in time t. However, when multiplied by $T_1/(\pi h)$ this factor is lower than one third of T_1.The higher the harmonic order, the lower the value of this factor. Even for the fundamental frequency component (h = 1), assuming T_1 and T are relatively close, the expression in brackets in (A-5) is between:

$$-T_1 - \frac{T_1}{3} \quad \text{and} \quad -T_1 + \frac{T_1}{3} \quad \text{(A-7)}$$

More importantly, the expression in brackets in (A-5) is always negative, regardless of the harmonic order, h; initial angle, α; and time of calculating the B-function, t.

Now let us turn our attention to the expression in (A-8).

$$sin\left(\frac{2\pi h}{T_1}T\right) \quad \text{(A-8)}$$

Assume T and T_1 are close. Therefore, factor (A-8) is negative if $T < T_1$ and positive if $T > T_1$. This is irrespective of the time and initial angle. For higher harmonics, the relationship can be reversed if T differs enough from T_1 (we will discuss this later in this appendix). Of course if $T = T_1$, (A-8) is exactly zero.

Knowing that the factor (A-7) is always negative, while the factor (A-8) is positive, zero, or negative depending on the relation between T and T_1, we conclude the following:

$$\begin{aligned} &\text{if } T < T_1 \text{ then B(t,T) is positive} \\ &\text{if } T = T_1 \text{ then B(t,T) is zero} \\ &\text{if } T > T_1 \text{ then B(t,T) is negative} \end{aligned} \quad \text{(A-9)}$$

As the input signal is the sum of all its harmonics, the B-function is the sum of the expressions in (A-5) for each of the harmonics. Each harmonic (A-1) follows the relationship in (A-9), and, therefore, the B-function for the input signal follows the relationship in (A-9).

The relationship in (A-9) is true for the fundamental frequency component and lower harmonics. Higher harmonics may have this relationship inverted. However, the magnitudes of higher harmonics are small compared with the fundamental in practical conditions. Observe, that the squared magnitudes appear in (A-5), making the harmonics with small magnitudes inconsequential in terms of (A-5). For example, assume 10 percent of the 7th harmonic. The contribution of this harmonic to the B-function for the input signal is $7 \cdot (0.1)^2$ = 0.07 pu, i.e., only 7 percent of the contribution of the fundamental frequency component. Even if the 7th harmonic violates (A-9), the B-function for the input signal with the fundamental will still follow (A-9) allowing the implementation of Section 4.

We can show that when using (4c) instead of (4a), the relationship in (A-9) is inverted.

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