Accurate Fault Locating During a Pole-Open Condition

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***Abstract*—Single- and double-ended fault location algorithms were among the first algorithms studied and devised in the field of numerical power network protection. These fault location methods, which have become conventional and are implemented in modern transmission line relays, have been recently supplemented with fault location based on traveling waves. Practically all the fault location methods assume a three-pole closed or normal mode of operation during the fault. The purpose of this paper is to demonstrate that errors will exist with the conventional single- or double-ended fault location calculation if the fault occurs during a pole-open condition, as could happen with single-pole reclosing. Compensated algorithms are presented that achieve accurate and error-free fault location under these conditions.**

I. Introduction

Single-ended fault location algorithms were among the first studied and developed at the onset of the digital protection technology era. These algorithms were also the first successfully implemented in numerical transmission line relays [1] [2]. Double-ended fault location algorithms followed, overcoming some of the shortcomings of earlier single-ended methods and achieving better accuracy [3] [4].

When applying single-pole tripping and reclosing schemes, a fault during a single-pole-open condition is rare but cannot be considered exceptional [5] [6]. This paper shows that conventional fault location calculations based on single- or double-ended data exhibit significant errors following a pole-open condition. Subsequently, this paper shows that compensated and corrected equations help achieve a level of accuracy equivalent to that found with a normal mode of operation (faults occurring with three poles closed). Recently, traveling wave-based fault location techniques have been introduced that achieve unprecedented accuracy with absolute distance accuracies better than 100 meters (300 feet) [7]. Although traveling wave-based fault location would maintain the same accuracy during a pole-open condition, the methods in this paper still use impedance-based techniques and process all six voltage and current waveforms from a single end or both ends of a transmission line.

The paper first reviews conventional single- and double-ended fault location principles and then introduces the required compensating formulas to take into account a pole-open condition.

II. Conventional Single-Ended Fault Location

A. Generic Equation of Distance Relaying

When applying distance protection to the elementary power network shown in Fig. 1, the apparent loop impedance (Z_{LP}) seen from the left bus and calculated for any of the six possible faulted loops (three for ground faults and three for phase faults) can be expressed by the following generic equation [8]:

$$Z_{LP} = \frac{V_{LP}}{I_{LP}} = d \bullet ZL1 + Rf \frac{K_{LP}}{K_I} \tag{1}$$

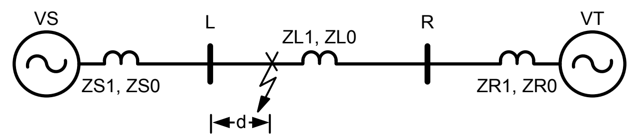

Fig. 1. Elementary single-line power system

In (1), V_{LP} and I_{LP} are the voltage and current relative to the particular faulted loop, *d* is the distance to the fault in per unit (pu) of the line length, ZL1 is the line positive-sequence impedance, Rf is the fault resistance, K_{LP} is a factor involving network parameters, and K_I is the ratio of the faulted loop current over the change of the same current. Reference [8] presents the voltage and current for any of the six possible faulted loops together with the corresponding K_{LP} factor. The location of the fault resistance (Rf) for the different fault loops is also shown in the same reference.

Using the demonstration of (1) for the particular case of the Phase-A-to-ground faulted loop in Reference [8], we end up with the following variables (defined for a relay installed on the left bus):

$$\begin{aligned} V_{LP} &= V_{AG} = VAL \\ I_{LP} &= I_{AG} = IAL + K_0 \bullet I0L \end{aligned} \tag{2}$$

$$K_{LP} = K_{AG} = \frac{3}{2C1 + C0\left(1 + K_0\right)} \tag{3}$$

$$K_I = \frac{I_{LP}}{I_{LP} - I_{LD}} = \frac{I_{LP}}{\Delta I_{LP}} = \frac{\left(IAL + K_0 \bullet I0L\right)}{\left(IAL + K_0 \bullet I0L\right) - I_{LD}} \tag{4}$$

In (2), K_0 is the zero-sequence compensation factor:

$$K_0 = \frac{ZL0 - ZL1}{ZL1} \tag{5}$$

In (3), C1 is the positive-sequence current distribution factor, seen from the left bus, relative to the network in Fig. 1. C2, the negative-sequence current distribution factor of the same network is equal to C1. C0 is the zero-sequence network current distribution factor. All three quantities are provided as:

$$C1 = C2 = \frac{\left(1 - d\right)ZL1 + ZR1}{ZL1 + ZS1 + ZR1} \tag{6}$$

$$C0 = \frac{(1-d)ZL0 + ZR0}{ZL0 + ZS0 + ZR0} \tag{7}$$

In (4), I_{LD} is the load or the prefault loop current. For a Phase-A-to-ground fault, the prefault current is:

$$I_{AG\_pf} = \frac{VS - VT}{ZL1 + ZS1 + ZR1} \tag{8}$$

B. Takagi Principle for Fault Location

Starting with (1), the Takagi principle for single-ended fault location can be defined by expressing the voltage of a particular impedance loop as follows [1]:

$$V_{LP} = d \bullet ZL1 \bullet I_{LP} + Rf \bullet K_{LP}\left(I_{LP} - I_{LD}\right) \tag{9}$$

The Takagi principle for single-ended fault location assumes that the factor K_{LP} is a pure real number, so if we multiply both sides of the equation by the conjugate of the difference current ($I_{LP} - I_{LD}$) and equate the imaginary parts of both sides of the equation, we have:

$$\mathrm{Im}\left(Rf \bullet K_{LP} \bullet (\Delta I_{LP})^2\right) = 0 \tag{10}$$

We end up with the following expression for the distance to the fault:

$$d = \frac{\mathrm{Im}\left[V_{LP} \bullet \mathrm{conj}(\Delta I_{LP})\right]}{\mathrm{Im}\left[ZL1 \bullet I_{LP} \bullet \mathrm{conj}(\Delta I_{LP})\right]} \tag{11}$$

C. Modified Takagi Method

The Takagi method as expressed in (10) and (11) could be fraught with errors because of the assumption that the factor K_{LP} is a real number. A correction factor can be introduced by defining a tilt angle, θ, in (12). This tilt angle compensates for the phase angle of the factor K_{LP} that was neglected in the first place. First, we express the complex number K_{LP} in polar form:

$$K_{LP} = |K_{LP}| \bullet e^{j\theta} \tag{12}$$

We multiply both sides of (9) by the next quantity and equate the imaginary parts of both sides of the new equation:

$$\mathrm{conj}\left(\Delta I_{LP} \bullet e^{j\theta}\right) \tag{13}$$

The following identity is applicable:

$$\mathrm{Im}\left[Rf \bullet |K_{LP}| \bullet e^{j\theta} \bullet \Delta I_{LP} \bullet \mathrm{conj}\left(\Delta I_{LP} \bullet e^{j\theta}\right)\right] = 0 \tag{14}$$

We end up therefore with the expression (15) for the distance to the fault:

$$d = \frac{\mathrm{Im}\left[V_{LP} \bullet \mathrm{conj}\left(\Delta I_{LP} \bullet e^{j\theta}\right)\right]}{\mathrm{Im}\left[ZL1 \bullet I_{LP} \bullet \mathrm{conj}\left(\Delta I_{LP} \bullet e^{j\theta}\right)\right]} \tag{15}$$

In (15), the angle θ is, in reality, a function of the distance (d) to the fault. If this function is known (and that implies an accurate knowledge of all the network impedances, particularly the source impedances), an iterative approach can be used to solve it. Typically, a simplified approach is taken where a single value of θ is used for the entire range of the distance (d) (varying from 0 to 1).

D. Eliminating the Prefault Current

For certain fault types, such as single-phase-to-ground faults, it is possible to eliminate the prefault current by expressing the voltage of the faulted impedance loop as a function of the total current at the fault [8]:

$$V_{AG} = d \bullet ZL1 \bullet I_{AG} + 3Rf \bullet I1F \tag{16}$$

For single Phase-A-to-ground faults, we have at the fault location for the total sequence currents [8]:

$$I1F = I2F = I0F \tag{17}$$

The negative- and zero-sequence currents at the relay can be expressed as the following functions of the total negative- and zero-sequence currents at the fault:

$$\begin{aligned} I2L &= C2 \bullet I2F \\ I0L &= C0 \bullet I0F \end{aligned} \tag{18}$$

Equation (16) can now be expressed as a function of the zero-sequence current at the relay:

$$V_{AG} = d \bullet ZL1 \bullet I_{AG} + 3Rf \bullet \frac{I0L}{C0} \tag{19}$$

The zero-sequence current distribution factor can be expressed using its modulus and argument:

$$C0 = |C0| \bullet e^{j\phi_0} \tag{20}$$

Equation (19) can now be expressed as:

$$V_{AG} = d \bullet ZL1 \bullet I_{AG} + 3Rf \bullet \frac{I0L}{|C0| \bullet e^{j\phi_0}} \tag{21}$$

We multiply both sides of (21) by the following expression and equate the imaginary parts of both sides:

$$\mathrm{conj}\left(I0L \bullet e^{-j\phi_0}\right) \tag{22}$$

The distance to the fault can finally be expressed as:

$$d = \frac{\mathrm{Im}\left[V_{AG} \bullet \mathrm{conj}\left(I0L \bullet e^{-j\phi_0}\right)\right]}{\mathrm{Im}\left[ZL1 \bullet I_{AG} \bullet \mathrm{conj}\left(I0L \bullet e^{-j\phi_0}\right)\right]} \tag{23}$$

In order to obtain (23), the following identity has been used:

$$\mathrm{Im}\left(\frac{I0L \bullet \mathrm{conj}\left(I0L \bullet e^{-j\phi_0}\right)}{|C0| \bullet e^{j\phi_0}}\right) = \mathrm{Im}\left(\frac{I0L^2}{|C0|}\right) = 0 \tag{24}$$

In (23), we replaced the prefault current with the zero-sequence current measured at the relay location during the fault [2]. As for (15), the compensating tilt angle provides an exact solution only for the single distance point for which it was calculated.

The same reasoning can be applied using the negative-sequence current so that we get an equivalent equation:

$$d = \frac{\mathrm{Im}\left[V_{AG} \bullet \mathrm{conj}\left(I2L \bullet e^{-j\phi_2}\right)\right]}{\mathrm{Im}\left[ZL1 \bullet I_{AG} \bullet \mathrm{conj}\left(I2L \bullet e^{-j\phi_2}\right)\right]} \tag{25}$$

Equation (23) with the zero-sequence current polarization and (25) with the negative-sequence current polarization are commonly used for single-ended fault location of Phase-A-to-ground faults. Equivalent expressions are available for Phase-B- or C-to-ground faults by introducing the proper loop voltage and current.

III. Pole-Open, Single-Ended Fault Location

A. Single-Ended Fault Location for Phase-A-to-Ground Fault During Phase B Pole-Open Condition

The Appendix demonstrates that if a Phase-A-to-ground fault occurs during a Phase B pole-open condition, the relation between the voltage and current for the faulted loop, distance to the fault, fault resistance, and total sequence current at the fault point is provided by:

$$VAL = d \bullet ZL1 \bullet \left(IAL + K_0 \bullet I0L\right) + 3Rf \bullet IF \qquad (26)$$

In (26), *IF* is the total sequence current at the fault point, as shown in Fig. 16 in the Appendix.

Equation (26) for the pole-open condition is similar to (16) for a normal mode (or three-pole closed) condition. This indicates that a distance element will have practically the same performance whether in a pole-open or normal mode of operation. The major difference in a pole-open condition is that the sequence currents can no longer be expressed as in (18). The relationship between the zero-sequence current at the relay location and the total zero-sequence current at the fault point is given by (27):

$$I0L = \frac{-a^2 \Delta V}{m+2n} - \frac{m_1 + 2n_1}{m+2n} IF \qquad (27)$$

Similarly, the relationship between the negative-sequence current at the relay and the total sequence current at the fault point is:

$$I2L = \frac{-a\Delta Vn}{m\left(m+2n\right)} + \left(\frac{a^2}{2}\frac{m_1+2n_1}{m+2n} - \frac{m_1\left(1-a\right)}{2m}\right) IF \qquad (28)$$

Finally, the relationship for the positive-sequence current is:

$$I1L = \frac{\Delta V\left(m+n\right)}{m\left(m+2n\right)} + \left(\frac{a}{2}\frac{m_1+2n_1}{m+2n} + \frac{m_1\left(a^2-1\right)}{2m}\right) IF \qquad (29)$$

For the elementary power network of Fig. 1, the expressions of m, n, m_1, and n_1 as functions of the network impedances are provided in the Appendix and are repeated here for convenience:

$$\begin{aligned} m &= ZL1 + ZS1 + ZR1 = ZL2 + ZS2 + ZR2 \\ n &= ZL0 + ZS0 + ZR0 \\ m_1 &= -\left(1-d\right)ZL1 - ZR1 \\ n_1 &= -\left(1-d\right)ZL0 - ZR0 \end{aligned} \qquad (30)$$

The following prefault sequence currents have been defined in the Appendix for a Phase B pole-open condition:

$$I0L_{pref} = \frac{-a^2\Delta V}{m+2n} \qquad (31)$$

$$I2L_{pref} = \frac{-a\Delta Vn}{m\left(m+2n\right)} \qquad (32)$$

$$I1L_{pref} = \frac{\Delta V\left(m+n\right)}{m\left(m+2n\right)} \qquad (33)$$

We now define the following set of current distribution factors in a Phase B pole-open condition:

$$C0_{pob} = \left|C0_{pob}\right| \bullet e^{j\psi_0} = -\frac{m_1+2n_1}{m+2n} \qquad (34)$$

$$C2_{pob} = \left|C2_{pob}\right| \bullet e^{j\psi_2} = \left(\frac{a^2}{2}\frac{m_1+2n_1}{m+2n} - \frac{m_1\left(1-a\right)}{2m}\right) \qquad (35)$$

$$C1_{pob} = \left|C1_{pob}\right| \bullet e^{j\psi_1} = \left(\frac{a}{2}\frac{m_1+2n_1}{m+2n} + \frac{m_1\left(a^2-1\right)}{2m}\right) \qquad (36)$$

Using (27), we express the total sequence current at the fault as a function of the zero-sequence current change at the relay [9]:

$$IF = \frac{I0L - I0L_{pref}}{C0_{pob}} = \frac{\Delta I0L}{C0_{pob}} \qquad (37)$$

The same can be done with respect to the positive- and negative-sequence current changes at the relay:

$$IF = \frac{\Delta I2L}{C2_{pob}} = \frac{\Delta I1L}{C1_{pob}} \qquad (38)$$

Replacing *IF* in (26) by its function of the change of the zero-sequence current at the relay, we get:

$$VAL = d \bullet ZL1 \bullet \left(IAL + K_0 \bullet I0L\right) + 3Rf \bullet \frac{\Delta I0L}{\left|C0_{pob}\right| \bullet e^{j\psi_0}} \qquad (39)$$

We can then multiply both sides of (39) by the conjugate of $\left(\Delta I0L \bullet e^{-j\psi_0}\right)$ and make use of the following identity:

$$Im\left(\frac{\Delta I0L \bullet conj\left(\Delta I0L \bullet e^{-j\psi_0}\right)}{\left|C0_{pob}\right| \bullet e^{j\psi_0}}\right) = Im\left(\frac{\Delta I0L^2}{\left|C0_{pob}\right|}\right) = 0 \qquad (40)$$

By then equating the imaginary parts of both sides of the equal sign, we derive (41) for the distance to the fault:

$$d = \frac{Im\left[VAL \bullet conj\left(\Delta I0L \bullet e^{-j\psi_0}\right)\right]}{Im\left[ZL1 \bullet I_{AG} \bullet conj\left(\Delta I0L \bullet e^{-j\psi_0}\right)\right]} \qquad (41)$$

The equation for the distance (d) to the fault during a pole-open condition using the zero-sequence current as the polarizing quantity has to be compared to (23) in normal mode. The main difference is that for a pole-open condition, we have to use the incremental quantity of the zero-sequence current, i.e., we have to subtract the prefault zero-sequence current existing during the pole-open condition from the zero-sequence current during the fault. Furthermore, the current distribution factor phase angle needs to be calculated, and this calculation requires the impedances of both the positive- and zero-sequence networks, as expressed in (34).

Alternatively, we could use the negative-sequence current as the polarizing quantity and get a similar equation for the distance to the fault:

$$d = \frac{\mathrm{Im}\left[\mathrm{VAL} \bullet \mathrm{conj}\left(\Delta \mathrm{I2L} \bullet e^{-j\psi_2}\right)\right]}{\mathrm{Im}\left[\mathrm{ZL1} \bullet \mathrm{I}_{\mathrm{AG}} \bullet \mathrm{conj}\left(\Delta \mathrm{I2L} \bullet e^{-j\psi_2}\right)\right]} \tag{42}$$

Finally, the same reasoning can be made with the positive-sequence current at the relay:

$$d = \frac{\mathrm{Im}\left[\mathrm{VAL} \bullet \mathrm{conj}\left(\Delta \mathrm{I1L} \bullet e^{-j\psi_1}\right)\right]}{\mathrm{Im}\left[\mathrm{ZL1} \bullet \mathrm{I}_{\mathrm{AG}} \bullet \mathrm{conj}\left(\Delta \mathrm{I1L} \bullet e^{-j\psi_1}\right)\right]} \tag{43}$$

We can use any one of the three equations, (41), (42), or (43), to provide the distance to the fault with data from one terminal. Each equation requires that the prefault sequence current be memorized ([9]) and that a compensating tilt angle be selected. In order to do so, the current distribution factor relative to each sequence current as provided by (34), (35), or (36) has to be investigated to select the best choice. A practical example is provided in the following subsection.

B. Practical Example of Fault Location

For this example, we test the single-ended fault location method on the 60 km, 120 kV line system shown in Fig. 2. A single Phase-A-to-ground fault is applied at time t = 100 ms, at 66.66 percent of the line length with Phase B open and a primary fault resistance of 50 ohms. We assume that the fault location is performed with single-ended data from the left terminal. First, the normal mode fault location using (23) and (25) are tested and then pole-open equations (41), (42), and (43) are used.

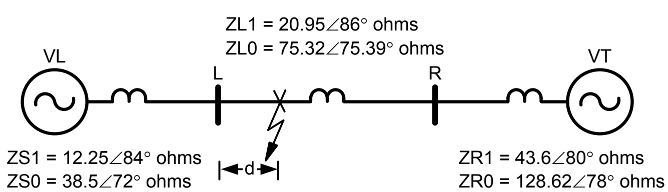

Fig. 2. 60 km line in 120 kV system

In all presented cases, the voltage and current waveforms are acquired at a rate of 16 samples per cycle (or 960 Hz) and processed through a full-cycle cosine filter in order to get the corresponding phasors.

1) Fault Location of Phase A-to-Ground Fault With Phase B Pole Open and Normal Mode Equations With Single Tilt Angle

The angles ϕ_2 and ϕ_0, relative to the current distribution factors C1 and C0 in normal mode, are functions of the distance (d) to the fault. They are plotted in Fig. 3 for the network of Fig. 2 and for a relay located at the left terminal. In order to compute the distance (d) to the fault using (23) and (25), we select a single tilt angle corresponding to the phase angle of C1 and C0 at the mid-distance, or d = 0.5 pu. We then use:

$$\begin{aligned} \phi_0 &= 1.1721° \\ \phi_2 &= -1.1132° \end{aligned} \tag{44}$$

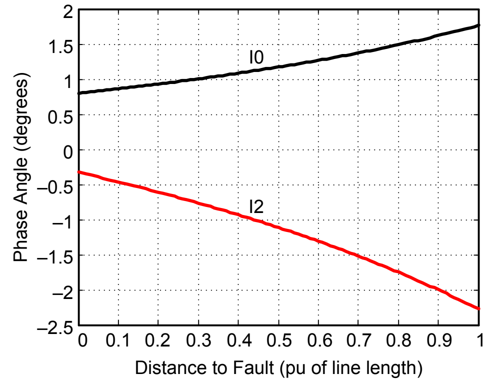

Fig. 3. Normal mode current distribution factors

Using the tilt angles provided by (44) and introducing these values in (23) and (25), we obtain (as shown in Fig. 4) the time loci of the distance (d) to the fault using either the negative- or zero-sequence currents as the polarizing quantities. From Fig. 4, we can see that whereas the zero-sequence polarization provides an error of about –46 percent (0.36 pu instead of 0.666 pu for the distance [d]), the negative-sequence polarization ends up with an error of 41 percent (fault location at 0.94 pu instead of 0.666 pu). The reason for this discrepancy could be explained by way of Fig. 5, where the magnitudes of the negative- and zero-sequence currents have been plotted before and after the fault. We can see that both the zero- and negative-sequence currents are significant before the fault.

Equations (25) and (42) are similar, with negative-sequence current as polarization for the fault location in normal and pole-open modes, respectively. So are (23) and (41), with the zero-sequence current polarization. The major difference is that in pole-open mode, we have to subtract the prefault sequence current existing during the pole-open condition. If it turns out that the prefault sequence current is small or negligible, the two equations should provide similar results for a pole-open condition. This example demonstrates that the normal mode single-ended fault location techniques will provide substantial errors in pole-open mode unless the prefault sequence current is small with respect to the same sequence current during the fault. When the prefault sequence current is negligible, the discrepancy between the two calculations is mainly due to the difference between the distribution factor phase angles.

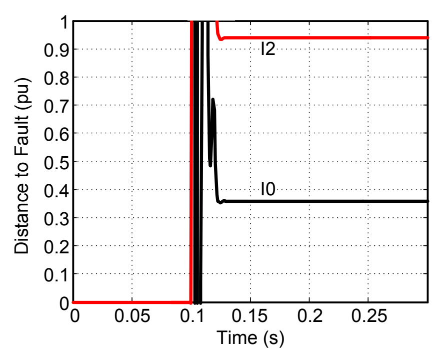

Fig. 4. Single-ended fault location during Phase B open, Phase-A-to-ground fault at 66.6 percent of the line length for Fig. 2 network using normal mode equations

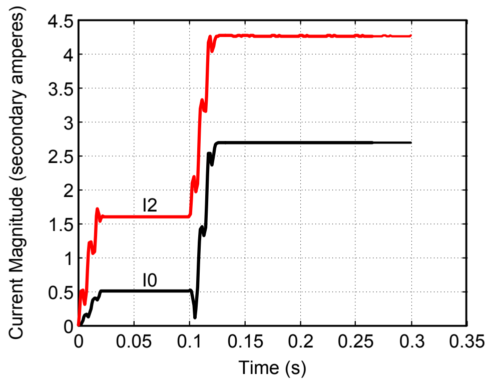

Fig. 5. Secondary negative- and zero-sequence current magnitudes

2) Fault Location With Pole-Open Equations and Single Tilt Angle

When using the pole-open fault location in (41), (42), and (44), Fig. 6 represents the variation of the current distribution factor phase angles corresponding to (34), (35), and (36), using the zero-, negative-, and positive-sequence current as the polarizing quantity, respectively. Looking at Fig. 6, it appears that the positive-sequence current would provide the best result because it has the least variation of the phase angle over the distance range of 0 to 1 pu, followed by the zero-sequence current, and then followed by the negative-sequence current, which has the largest variation of the phase angle. In each case, we select the tilt angle corresponding to the mid-range distance value of 0.5 pu, as provided by (45):

$$\begin{aligned}\psi_1 &= -0.0836^\circ\\ \psi_2 &= -1.1826^\circ\\ \psi_3 &= 0.8721^\circ\end{aligned} \tag{45}$$

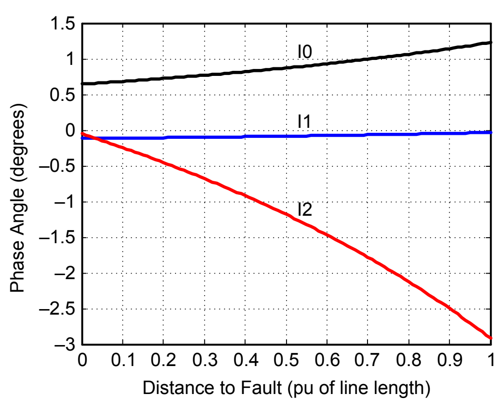

Fig. 6. Current distribution factor phase angles for a Phase B pole-open condition for the Fig. 2 network

Fig. 7 shows the fault location in pu of the line length. The zero-sequence current polarization provides the best result, although the positive-sequence current polarization fault location is very close. The polarization using the negative-sequence current provides the worst fault location relative average error, although it is still below 2.3 percent (0.6807 pu measured against the true value of 0.666 pu). The zero-sequence current polarization has an average relative error of 0.3 percent (0.668 pu measured). The positive-sequence current polarization has an average error of 0.9 percent (0.672 pu measured).

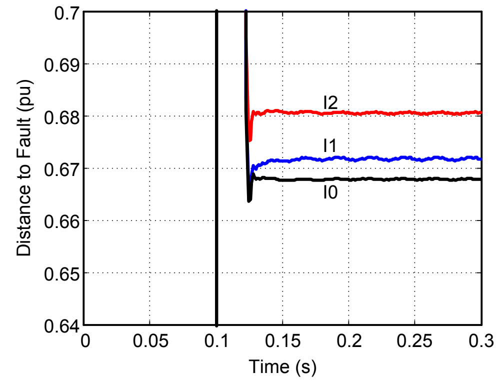

Fig. 7. Single-ended fault location during a Phase B pole-open condition, Phase-A-to-ground fault at 66.6 percent of the line length for the Fig. 2 network

3) Fault Location With Pole-Open Equations and Polynomial Approximation of Tilt Angle

In Fig. 6, the phase angle of the negative-sequence current distribution factor as a function of the distance (d) to the fault can be curve-fitted with high accuracy using a third order polynomial:

$$\psi_2(d) = -0.5015 \bullet d^3 - 0.4387 \bullet d^2 - 1.9232 \bullet d - 0.0475 \tag{46}$$

The distance to the fault can also be computed using (42), where the tilt angle is now a function of the distance to the fault:

$$d = \frac{\mathrm{Im}\left[VAL \bullet \mathrm{conj}(\Delta I2L) \bullet e^{-j\psi_2(d)}\right]}{\mathrm{Im}\left[ZL1 \bullet I_{AG} \bullet \mathrm{conj}(\Delta I2L) \bullet e^{-j\psi_2(d)}\right]} \tag{47}$$

The idea is to use an iterative process starting with a tilt angle of zero first and then calculating the distance (d). Then, we use the newly calculated distance (d) to compute a new tilt angle using the polynomial curve-fit equation (46), and the process is repeated until the error is within a predefined tolerance. Fig. 8 shows the loci of the computed distance to the fault with respect to time. For this simulation, six iterations gave the required error tolerance. The iterative method allows a distance to be calculated with practically no error as opposed to the method where a single mid-curve tilt angle is used.

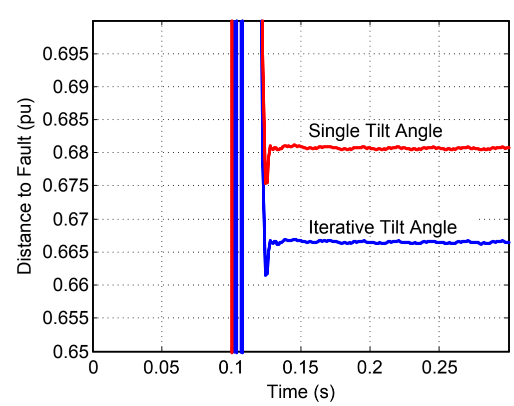

Fig. 8. Iterative fault location with polynomial approximation of the tilt angle using the negative-sequence current polarization

We must realize that the iterative fault location method is a bit unrealistic because it requires a precise knowledge of the network parameters and particularly the source impedances. This is something that is not easily achievable in reality with a practical application. The example, however, illustrates the factors that impact the single-ended fault location and shows what is possible with an ideal single-ended fault location method.

IV. Double-Ended Fault Location

A. Double-Ended Fault Location in Normal Mode

Double-ended fault location is typically applied by using the negative-sequence network in normal mode, as represented in Fig. 9 [3] [4]. Assuming synchronous sampling at both line terminals, we can write the following equation based on the property that the voltage drop between the left and right bus and V_F is the same [4]:

$$V2L - I2L \cdot d \cdot ZL1 = V2R - (1-d) \cdot ZL1 \cdot I2R \tag{48}$$

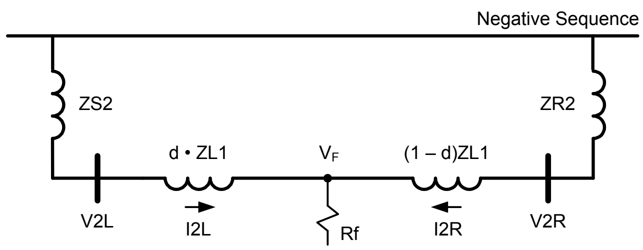

Fig. 9. Negative-sequence network

From (48), we derive the fault location (d):

$$d = \frac{(V2L - V2R) + ZL1 \cdot I2R}{ZL1 \cdot (I2L + I2R)} \tag{49}$$

B. Double-Ended Fault Location With Phase B Open

The same principle can be applied in a pole-open condition. From the negative-sequence network shown in Fig. 16 (in the Appendix), we can write:

$$\begin{aligned} &V2L - 0.33 \cdot a^2 \cdot V_{XYB} - I2L \cdot d \cdot ZL1 = \\ &V2R - (1-d) \cdot ZL1 \cdot I2R \end{aligned} \tag{50}$$

From the positive-sequence network in Fig. 16, we can write:

$$\begin{aligned} &V1L - 0.33 \cdot a \cdot V_{XYB} - I1L \cdot d \cdot ZL1 = \\ &V1R - (1-d) \cdot ZL1 \cdot I1R \end{aligned} \tag{51}$$

From (51), we can extract the expression of V_{XYB}:

$$\begin{aligned} &-0.33 \cdot a \cdot V_{XYB} = \\ &(V1R - V1L) + d \cdot ZL1(I1L + I1R) - ZL1 \cdot I1R \end{aligned} \tag{52}$$

Multiplying both sides of (52) by *a* and replacing ($-0.33 \cdot a^2 \cdot V_{XYB}$) in (50) by its expression derived from (52), we get the distance to the fault (d) during the pole-open condition for Phase B, using the double-ended method.

$$d = \frac{(V2L - V2R) + ZL1(I2R - a \cdot I1R) - a(V1L - V1R)}{ZL1(I2L + I2R) - a \cdot ZL1(I1L + I1R)} \tag{53}$$

Comparing (49) during the normal condition to (53) for the Phase B pole-open condition, we can see that (53) now involves voltages and currents from both the positive- and negative-sequence networks.

C. Double-Ended Fault Location With Phase A Open

Following the same reasoning as in the previous subsection, the double-ended fault location during a Phase A pole-open condition can be expressed as:

$$d = \frac{(V2L - V2R) + ZL1(I2R - I1R) - (V1L - V1R)}{ZL1(I2L + I2R) - ZL1(I1L + I1R)} \tag{54}$$

D. Double-Ended Fault Location With Phase C Open

Similarly, the double-ended fault location method for a Phase C pole-open condition can be expressed as:

$$d = \frac{(V2L - V2R) + ZL1(I2R - a^2 \cdot I1R) - a^2(V1L - V1R)}{ZL1(I2L + I2R) - a^2 \cdot ZL1(I1L + I1R)} \tag{55}$$

E. Examples of Application

We first test the double-ended fault location on the 120 kV system of Fig. 2 and then on the longer 500 kV system of Fig. 10. Both the normal mode (49) and the pole-open mode (53) and (55) equations are tested.

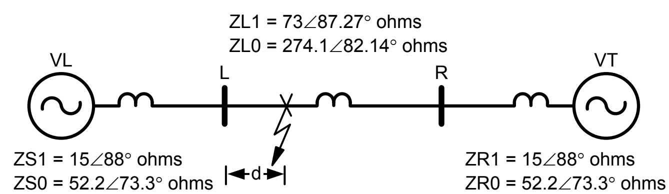

Fig. 10. 200 km, 500 kV line system

1) Phase-C-to-Ground Fault With Phase B Open at 66 Percent of the Line Length With Fault Resistance of 20 Ohms on 120 kV System of Fig. 2

Fig. 11 shows the time trajectories of the distance (d) to the fault using both the normal mode (49) and the pole-open mode (53) equations. Obviously, the pole-open mode equation is more accurate than the normal mode equation (0.921 pu measured against the true value of 0.666 pu). In Fig. 12, the vertical axis provides the distance to the fault in pu of the line length. This axis has been expanded to further demonstrate the almost perfect accuracy of the pole-open method (the actual fault location is at 66.6 percent of the line length [0.666 pu]).

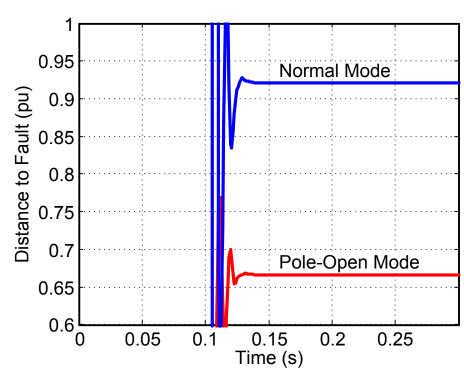

Fig. 11. 120 kV line double-ended fault location for Phase-C-to-ground fault with Phase B open and fault resistance of 20 ohms

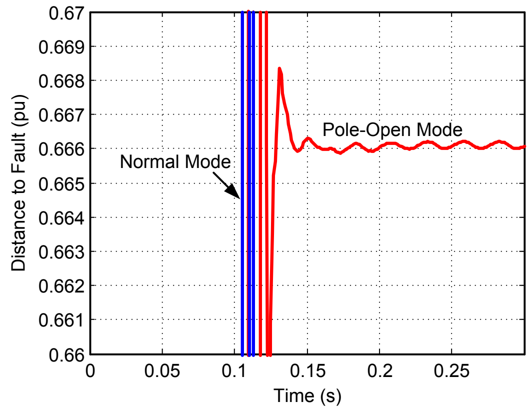

Fig. 12. Fig. 11 with expanded vertical scale

2) Phase A-to-Ground Fault With Phase C Open at 33.3 Percent of the Line Length With Fault Resistance of 10 Ohms on 500 kV System of Fig. 10

Fig. 13 shows again the loci of the distance (d) to the fault using both the normal mode (49) and the pole-open mode (55) equations. In Fig. 14, the vertical scale provides the distance to the fault in pu of the line length. It has been expanded to demonstrate the accuracy of the dual-ended pole-open method. The relative error is approximately 0.5 percent.

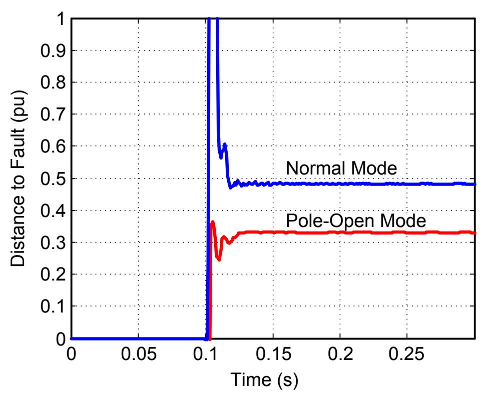

Fig. 13. 500 kV line double-ended fault location for Phase A-to-ground fault with Phase C open and fault resistance of 10 ohms at 33.3 percent of line length

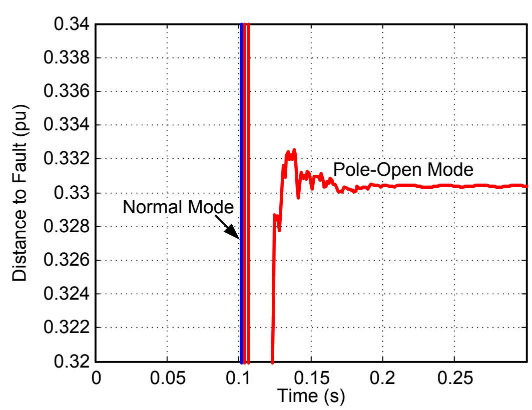

Fig. 14. Fig. 13 with expanded vertical scale

V. Conclusion

This paper shows that significant errors will be encountered when conventional single-ended fault location methods are used for fault location during a pole-open condition on a transmission line, particularly if the prefault sequence currents (negative or zero) are present. This situation becomes particularly true if the fault resistance increases so that the sequence current during the fault becomes comparable to the sequence current existing before the fault. The paper introduces equations with the proper compensations to take into account the sequence current existing before the fault.

The paper also shows that the conventional fault location equation based on double-ended data for one sequence network (typically negative sequence) needs to be compensated for by using the combined information from two sequence networks, positive and negative, in order to achieve accuracy comparable to the normal mode situation. Using information from two sequence networks allows the resolution of the new unknown corresponding to the voltage across the open breaker pole.

VI. Appendix: Resolution of an Internal Single Phase-A-to-Ground Fault With Phase B Open Using the Sequence Network

We consider the elementary network of Fig. 1. The sequence network corresponding to a Phase B pole-open condition is represented in Fig. 15.

In order to draw the sequence network for a Phase B pole-open condition, two constraints must be embedded into the network: first, a voltage constraint, and second, a current constraint. We assume that Phase B is open between two points, *x* and *y*. The sequence voltages between Points *x* and *y* are provided by the following conditions:

$$\begin{pmatrix} V1_{xy} \\ V2_{xy} \\ V0_{xy} \end{pmatrix} = \frac{1}{3} \bullet \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \bullet \begin{pmatrix} VA_{xy} \\ VB_{xy} \\ VC_{xy} \end{pmatrix} \tag{56}$$

In (56), *a* is the conventional complex operator $1\angle 120°$. VA_{xy} and VC_{xy} are clearly zero, so we end up with:

$$\begin{aligned} V1_{xy} &= \left(\frac{1}{3}\right) aVB_{xy} \\ V2_{xy} &= \left(\frac{1}{3}\right) a^2 VB_{xy} \\ V0_{xy} &= \left(\frac{1}{3}\right) VB_{xy} \end{aligned} \tag{57}$$

The current constraint is expressed by the condition that the Phase B current must be equal to zero or:

$$IB = a^2 I1L + aI2L + I0L = 0 \tag{58}$$

The three ideal transformers represented in the sequence network of Fig. 15 implement the two voltage and current constraints.

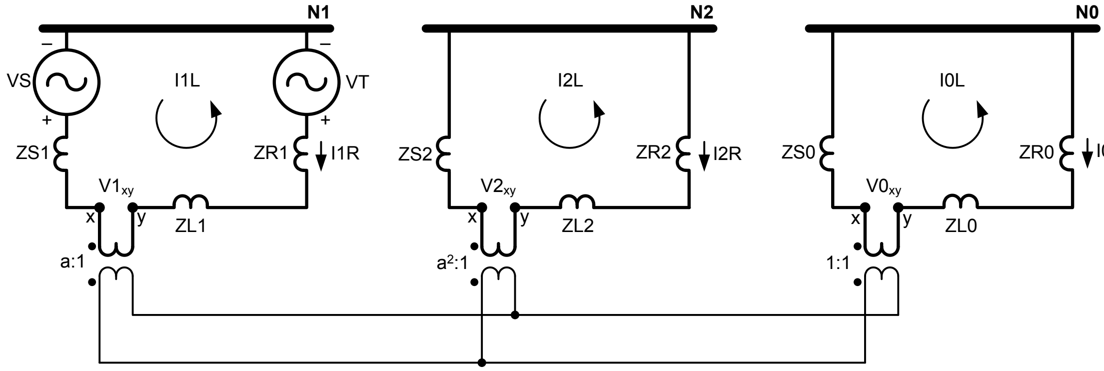

Fig. 15. Sequence network of Phase B pole-open condition elementary network

$$\begin{pmatrix} ZL1+ZS1+ZR1 & 0 & 0 & a \\ 0 & ZL1+ZS1+ZR1 & 0 & a^2 \\ 0 & 0 & ZL0+ZS0+ZR0 & 1 \\ a^2 & a & 1 & 0 \end{pmatrix} \bullet \begin{pmatrix} I1L \\ I2L \\ I0L \\ \left(\frac{1}{3}\right) VB_{xy} \end{pmatrix} = \begin{pmatrix} VL-VR \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{59}$$

The sequence network of Fig. 15 can be resolved for the unknown sequence currents by solving the linear matrix equation (59).

We define the following variables:

$$\begin{aligned} \Delta V &= VS - VT \\ m &= ZL1 + ZS1 + ZR1 = ZL2 + ZS2 + ZR2 \\ n &= ZL0 + ZS0 + ZR0 \\ m_1 &= -(1-d)ZL1 - ZR1 \\ n_1 &= -(1-d)ZL0 - ZR0 \\ p &= m + m_1 = dZL1 + ZS1 = dZL2 + ZS2 \\ q &= n + n_1 = dZL0 + ZS0 = dZL0 + ZS0 \end{aligned} \tag{60}$$

Using the Gaussian elimination process, we can resolve the system of equations in (59), which leads to the solution for the three sequence currents as:

$$I1L_{preflt} = -I1R_{preflt} = \frac{\Delta V(m+n)}{m(m+2n)} \tag{61}$$

$$I2L_{preflt} = -I2R_{preflt} = -\frac{a\Delta Vn}{m(m+2n)} \tag{62}$$

$$I0L_{preflt} = -I0R_{preflt} = -\frac{a^2\Delta V}{m+2n} \tag{63}$$

It should be borne in mind at this stage of the analysis that the three sequence currents determined during the Phase B pole-open condition constitute the prefault currents before any other fault occurs at a later stage. For this reason, the sequence currents are shown in (61) through (63) with a prefault (preflt) subscript.

Following the same reasoning, the Phase A prefault current on the left side can be computed as the sum of all three sequence currents:

$$IAL_{preflt} = \frac{m\Delta V(1-a^2) + n\Delta V(1-a)}{m(m+2n)} \tag{64}$$

We now assume that during the Phase B pole-open condition, a subsequent Phase-A-to-ground fault occurred at a distance (d) from the left bus of the line. This faulted sequence network is represented in Fig. 16, where the new condition of a Phase-A-to-ground fault has been added to the already represented Phase B pole-open condition. The sequence currents can now be resolved by solving the linear equations in (65) and (66).

By using the same Gaussian elimination process as (65), the current at the fault (IF) and the sequence currents at the left-hand-side line bus can be computed as (66).

$$\begin{pmatrix} ZL1+ZS1+ZR1 & 0 & 0 & -(1-d)ZL1-ZR1 & a \\ 0 & ZL2+ZS2+ZR2 & 0 & -(1-d)ZL2-ZR2 & a^2 \\ 0 & 0 & ZL0+ZS0+ZR0 & -(1-d)ZL0-ZR0 & 1 \\ ZS1+dZL1 & ZS2+dZL2 & ZS0+dZL0 & 3R & 0 \\ a^2 & a & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I1L \\ I2L \\ I0L \\ IF \\ \left(\frac{1}{3}\right)VB_{xy} \end{pmatrix} = \begin{pmatrix} \Delta V \\ 0 \\ 0 \\ VA \\ 0 \end{pmatrix} \tag{65}$$

$$IF = \frac{-2a^2 m\Delta V(-0.5p-q) - \Delta Vp(1-a)(m+2n) + 2mVA(m+2n)}{2m(-0.5p-q)(m_1+2n_1) - 3pm_1(m+2n) + 6Rf\ m(m+2n)} \tag{66}$$

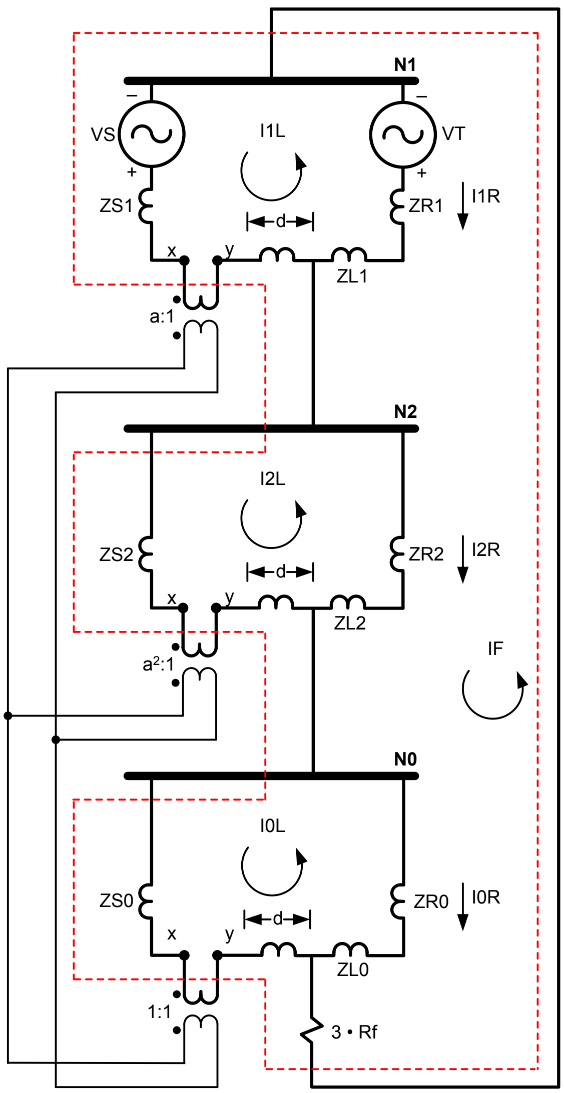

Fig. 16. Sequence network of Phase B pole-open condition with Phase-A-to-ground fault elementary network

$$I0L = \frac{-a^2\Delta V}{m+2n} - \frac{m_1+2n_1}{m+2n} IF \tag{67}$$

$$I2L = \frac{-a\Delta Vn}{m(m+2n)} + \left(\frac{a^2}{2}\frac{m_1+2n_1}{m+2n} - \frac{m_1(1-a)}{2m}\right) IF \tag{68}$$

$$I1L = \frac{\Delta V(m+n)}{m(m+2n)} + \left(\frac{a}{2}\frac{m_1+2n_1}{m+2n} + \frac{m_1(a^2-1)}{2m}\right) IF \tag{69}$$

The Phase A fault current on the left side is equal to the sum of the three sequence currents:

$$\begin{aligned} IAL = {} & \frac{\Delta V(m+n)}{m(m+2n)} + \frac{-a\Delta Vn}{m(m+2n)} - \frac{a^2\Delta V}{m+2n} \\ & + \left(\left(\frac{a^2}{2}+\frac{a}{2}-1\right)\left(\frac{m_1+2n_1}{m+2n}\right) + \frac{m_1(a^2-2+a)}{2m}\right) IF \end{aligned} \tag{70}$$

Looking at (67) through (69) for the left-side bus and considering (61) through (63) for the sequence currents in the open-pole condition only, we can see that the sequence currents at both extremities of the lines are composed of a prefault term and a second term that is proportional to *IF*, so we can now write:

$$I0L = I0L_{preflt} - \frac{m_1+2n_1}{m+2n} IF \tag{71}$$

$$I2L = I2L_{preflt} + \left(\frac{a^2}{2}\frac{m_1+2n_1}{m+2n} - \frac{m_1(1-a)}{2m}\right) IF \tag{72}$$

$$I1L = I1L_{preflt} + \left(\frac{a}{2}\frac{m_1+2n_1}{m+2n} + \frac{m_1(a^2-1)}{2m}\right) IF \tag{73}$$

Following the same line of thinking, the left-side Phase A current can be expressed as:

$$\begin{aligned} IAL = {} & IAL_{preflt} \\ & + \left(\left(\frac{a^2}{2}+\frac{a}{2}-1\right)\left(\frac{m_1+2n_1}{m+2n}\right) + \frac{m_1(a^2-2+a)}{2m}\right) IF \end{aligned} \tag{74}$$

Following the identity:

$$a^2 + a = -1 \tag{75}$$

Equation (74) can be rewritten as:

$$IAL = IAL_{preflt} - \frac{3}{2}\left(\frac{m_1+2n_1}{m+2n} + \frac{m_1}{m}\right) IF \tag{76}$$

If we sum up the voltages around the loop shown as the red dashed line in Fig. 16, and taking into account that:

$$V1_{xy} + V2_{xy} + V0_{xy} = 0 \tag{77}$$

We end up with:

$$\begin{aligned}&\mathrm{V1L} + \mathrm{V2L} + \mathrm{V0L} - \mathrm{d} \bullet \mathrm{ZL1} \bullet \left(\mathrm{I1L} + \mathrm{I2L}\right) \\ &-\mathrm{d} \bullet \mathrm{ZL0} \bullet \mathrm{I0L} - 3\mathrm{Rf} \bullet \mathrm{IF} = 0\end{aligned} \tag{78}$$

If we add and subtract d • ZL1 • I0L, we get:

$$\begin{aligned}&\mathrm{V1L} + \mathrm{V2L} + \mathrm{V0L} - \mathrm{d} \bullet \mathrm{ZL1} \bullet \left(\mathrm{I1L} + \mathrm{I2L} + \mathrm{I0L}\right) \\ &-\mathrm{d} \bullet \mathrm{I0L}\left(\mathrm{ZL0} - \mathrm{ZL1}\right)\mathrm{I0L} - 3\mathrm{Rf} \bullet \mathrm{IF} = 0\end{aligned} \tag{79}$$

Because we have:

$$\begin{aligned}\mathrm{VAL} &= \mathrm{V1L} + \mathrm{V2L} + \mathrm{V0L} \\ \mathrm{IAL} &= \mathrm{I1L} + \mathrm{I2L} + \mathrm{I0L}\end{aligned} \tag{80}$$

We finally end up with:

$$\mathrm{VAL} = \mathrm{d} \bullet \mathrm{ZL1} \bullet \left(\mathrm{IAL} + \mathrm{K}_0 \bullet \mathrm{I0L}\right) + 3\mathrm{Rf} \bullet \mathrm{IF} \tag{81}$$

VII. Acknowledgements

The authors are indebted to Normann Fischer from Schweitzer Engineering Laboratories, Inc. for reviewing the paper.

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IX. Biographies

Gabriel Benmouyal, P.E., received his B.A.Sc. in electrical engineering and his M.A.Sc. in control engineering from Ecole Polytechnique, Université de Montréal, Canada. In 1978, he joined IREQ, where his main fields of activity were the application of microprocessors and digital techniques for substations and generating station control and protection systems. In 1997, he joined Schweitzer Engineering Laboratories, Inc. as a principal research engineer. Gabriel is a registered professional engineer in the Province of Québec and an IEEE senior member and has served on the Power System Relaying Committee since May 1989. He holds more than six patents and is the author or coauthor of several papers in the fields of signal processing and power network protection and control.

Brian Smyth received a B.S.E.E. and M.S.E.E. from Montana Tech at the University of Montana in 2006 and 2008, respectively. He joined Montana Tech as a visiting professor in 2008 and taught classes in electrical circuits, electric machinery, instrumentation and controls, and power system analysis. He joined Schweitzer Engineering Laboratories, Inc. in 2009 as an associate power engineer in the research and development division. Brian is a lead product engineer in the transmission and substation group. He is a coauthor of several papers on battery impedance measurement and power system out-of-step conditions. He is also an active IEEE member and a registered professional engineer in the state of Washington.

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