Analysis of an Autotransformer Restricted Earth Fault Application

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Abstract—Restricted earth fault, or zero-sequence differential protection, is beneficial in transformer applications. Because it does not respond to load current, it offers a significant improvement in sensitivity over traditional differential protection. Ground current in the transformer neutral is used as a reference and is compared to zero-sequence current at the terminals to determine if a fault is internal to the transformer. The predictability of the neutral current phase angle is critical to it being a stable reference. It is not well understood how the magnitude and direction of circulating zero-sequence current in a delta tertiary relates to the zero-sequence current in the autotransformer neutral. This technical paper derives that relationship, explains restricted earth fault protection, and uses a real-world, unexpected relay operation to demonstrate these concepts and make relay settings recommendations.

I. INTRODUCTION

Transformer differential protection is simple in concept and the primary means for protecting power transformers. A transformer differential relay compares current entering the transformer to current leaving the transformer. The phasor sum of the currents is the operate (or difference) current. If the currents are not nearly equal, we assume a possible internal fault.
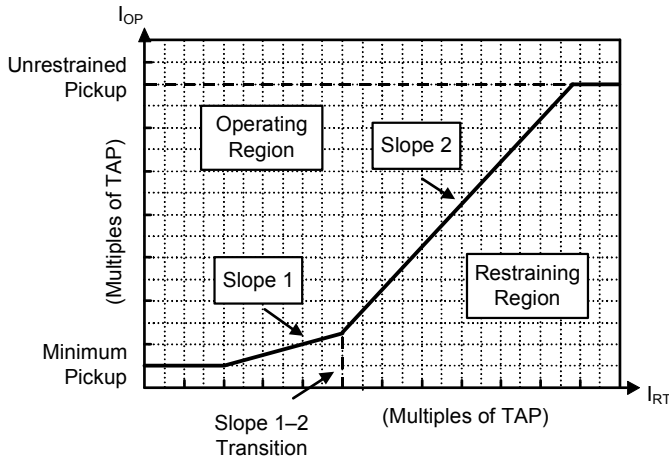

Fig. 1.   Percentage Restraint Differential Characteristic

Instead of comparing the operate current to a simple threshold or pickup, the relay calculates the ratio of operate current to a restraint quantity. Restraint current is typically calculated as the scalar sum of the CT currents, (i.e., the average of the current magnitudes). When the operate-to-restraint current ratio exceeds a certain percentage or slope, as shown in Fig. 1, the relay operates. The lowest horizontal portion of the trip threshold represents the minimum sensitivity of the relay. This minimum setting is typically

30 percent or 0.3 per unit of the transformer-rated load current and is necessary to address errors at very low current magnitudes, such as those due to tap changer operations. Because a CT error resulting from saturation is most likely to be a problem at higher current levels, the slope of the characteristic provides increased security as currents increase.

The slope characteristic of the primary phase-differential element causes relay sensitivity to decrease as load current increases. Therefore, at maximum transformer loading, the phase-differential element may be unable to detect ground faults near the neutral until the fault evolves. Because of the primary differential element's response to phase currents and the inherent security measures of the slope characteristic, restricted earth fault (REF), or zero-sequence differential protection, can provide better detection of phase-to-ground faults near the neutral on wye-connected windings. REF elements are immune to balanced load, so set these elements to be more sensitive for this type of fault. Ground current in the transformer neutral is used as a reference and is compared to zero-sequence current at the wye-connected winding terminals to determine if a ground fault is internal to the transformer-wye windings [1]. Section II describes in detail the operation of a REF element.

As with any differential or directional application, the phase angle measurement stability and the predictability of current direction are critical to successful operation. In autotransformer installations, it is not well understood how the magnitude and direction of circulating zero-sequence current in a delta tertiary relates to the zero-sequence current in the autotransformer neutral. Section III derives the mathematics and explains this relationship. From this, conclusions are drawn as to what applications are appropriate for using CTs within the delta tertiary versus a CT in the autotransformer neutral.

Section IV shares a real-world event report from an unexpected operation in which CTs in an autotransformer delta tertiary were used as the reference current for a REF application. Through detailed analysis, we develop a better understanding of the REF element operation, the zero-sequence currents observed in an autotransformer, and the lessons learned.

II. REF PROTECTION

REF elements are used to provide sensitive protection against ground faults in wye-connected transformer windings. The element is "restricted" in the sense that protection is

restricted to ground faults within a zone defined by neutral and line CT placement.

Because it employs a neutral CT at one end of the winding and the normal set of three CTs at the line end of the winding, REF protection can detect only ground faults within that particular wye-connected winding. For REF to function, the line-end CTs must also be connected in wye, because the technique uses zero-sequence current comparisons. Delta-connected CTs cancel out all zero-sequence components of the currents, eliminating one of the quantities the REF element needs for comparison.

REF implementations commonly use a directional element (32I) that compares the direction of an operating current, derived from the line-end CTs, with the polarizing current obtained from the neutral CT. A zero-sequence current threshold and positive-sequence restraint supervise tripping. Apply REF protection to a single wye winding in a transformer or to an entire autotransformer winding with as many as three sets of line-end CT inputs. The line-end, three-phase winding inputs can additionally be used for normal percentage-restraint differential, overcurrent protection, or metering purposes.
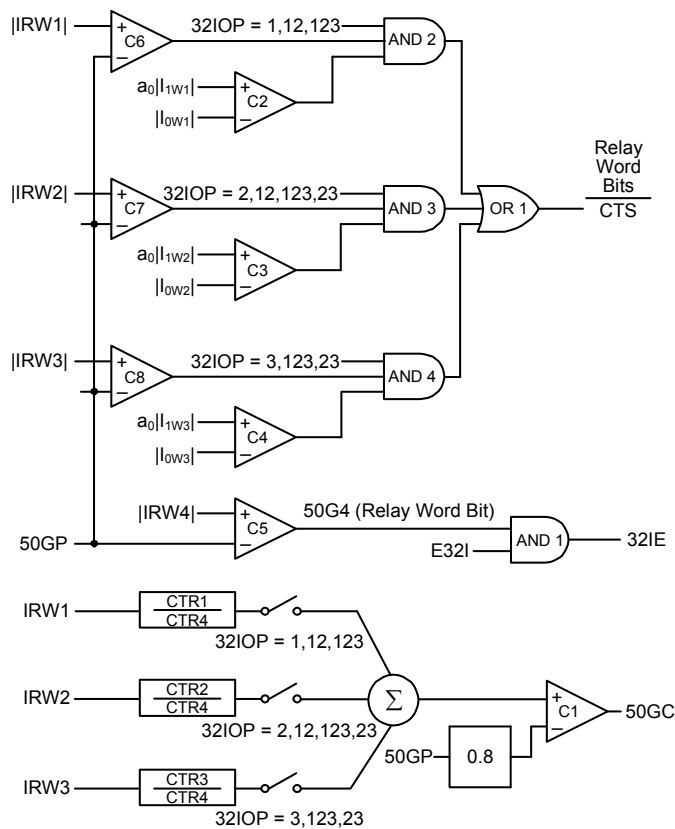

Fig. 2.   REF Enable/Block Logic

Fig. 2 shows the REF simplified enable/block logic. The upper logic group determines whether to enable the REF directional element by asserting the 32IE bit. The two enabling quantities are assertion of the E32I programmable logic equation and a magnitude of the neutral CT secondary current (IRW4) greater than the pickup setting 50GP. The topmost part of this logic is a blocking function. This function asserts if any of the winding residual currents used in the REF function are less than a positive-sequence current restraint factor (a_0) multiplied by the positive-sequence current for their respective winding. Such a winding residual current value might occur with "false I_0" or if zero-sequence current for that winding exceeds 50GP. False I_0 can occur in cases of CT saturation during heavy three-phase faults. If the blocking logic asserts, the CTS bit asserts. To prevent 32IE assertion when CTS asserts, set the E32I logic equation setting not equal to CTS.

The lower logic group adjusts the winding residual currents to a common sensitivity level with the neutral CT, calculates a phasor sum of the appropriate currents, and compares this sum to the 50GP pickup value. If the sum is greater than the pickup level, the 50GC bit asserts. This bit indicates that the winding currents are present in sufficient magnitude.

Fig. 3 illustrates the logic of the REF directional element 32I. At this stage, the element decides whether or not to operate.
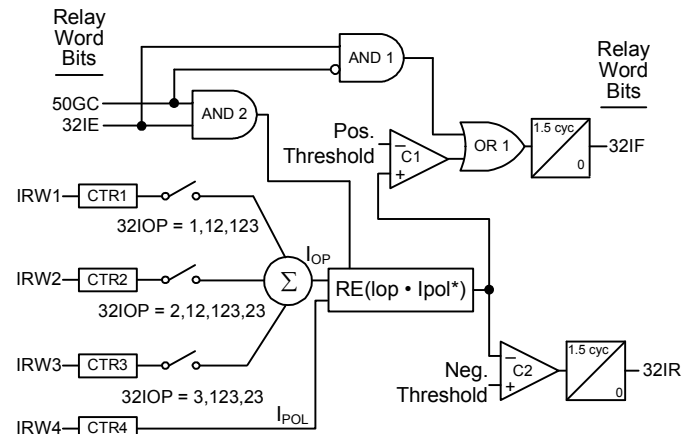

Fig. 3.   REF Directional Element

The relay enables the 32I directional element if the output of AND 2 in Fig. 3 asserts. This will occur if 32IE and 50GC assert.

The directional element compares the polarizing current to the operating current and indicates forward (internal) fault location or reverse (external) fault location. The forward/internal indication occurs if the fault is within the protected winding, between the line-end CTs and the neutral CT. The relay multiplies each current by the appropriate CT ratio to convert input currents to actual primary amperes. This must be done to properly sum the currents in the autotransformer winding.

The polarizing current (I_{POL}) is the neutral CT current multiplied by the neutral CT ratio (CTR4) to produce a primary current value. The operating current (I_{OP}) is the phasor sum of the winding residual currents, which is also on a primary basis. The 32IOP setting determines the appropriate IRWn (n = 1, 2, 3), which the relay multiplies by the associated CTRn. The relay then sums the products. The 32I element calculates the real part of I_{OP} times I_{POL}* (I_{POL} complex conjugate). This equates to $|I_{OP}|$ times $|I_{POL}|$ times the cosine of the angle between them. The result is positive if the

angle is within ±90 degrees, indicating a forward or internal fault. The result is negative if the angle is greater than +90 or less than –90 degrees, indicating a reverse or external fault. The relay compares the output of the 32I element to positive and negative thresholds to ensure security for very small currents or for an angle very near +90 or –90 degrees. If the 32I output exceeds the threshold test, then it must persist for at least 1.5 cycles before 32IF (forward) or 32IR (reverse) assert. 32IF assertion constitutes a decision to trip by the REF function.

A second path can also assert the 32IF bit. This path comes from AND 1 at the top of Fig. 3. The gate asserts if 32IE is asserted. This assertion indicates that neutral current is above pickup but 50GC is not asserted, indicating no line-end current flow. This logic covers the situation of an internal wye-winding fault with the line-end breaker open.

Tripping can be performed directly by the 32IF bit. If additional security is desired, the 32IF bit can be used to torque control an inverse-time curve for delayed tripping.

Fig. 4 shows the output of the REF protection function. Timing is on an extremely inverse-time overcurrent curve at the lowest time-dial setting, with 50GP as the pickup setting.
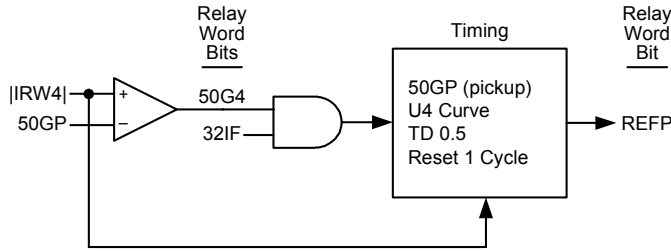

Fig. 4. REF Protection Output

Logic bit 32IF (forward fault) torque controls the timing curve, while IRW4 operates the timing function. The curve resets in one cycle if the current drops below pickup or if 32IF deasserts. When the curve times out, logic bit REFP asserts. Tripping can be performed directly by logic bit REFP [2].

III. AUTOTRANSFORMER ZERO-SEQUENCE FAULT CURRENT DERIVATION

An important part of REF protection is having a reliable zero-sequence source for polarizing current. For a two-winding, delta-wye transformer, the current in the X0 (or H0) bushing is the only ground source available. An autotransformer with a delta-tertiary winding provides zero-sequence polarizing current in two locations:

- Neutral of the transformer
- Inside the delta-tertiary winding

This section describes the appropriate location in an autotransformer to use a REF scheme polarizing quantity and how the two currents are affected by changes in source and transformer impedances.

Consider the example system in Fig. 5, with a ground fault placed on the high-voltage (HV) system. The zero-sequence currents present in the transformer neutral and tertiary windings are given in (1) and (2), respectively.

a) One-Line Diagram

100 MVA
VH = 138 kV
VM = 69 kV
VL = 13.2 kV

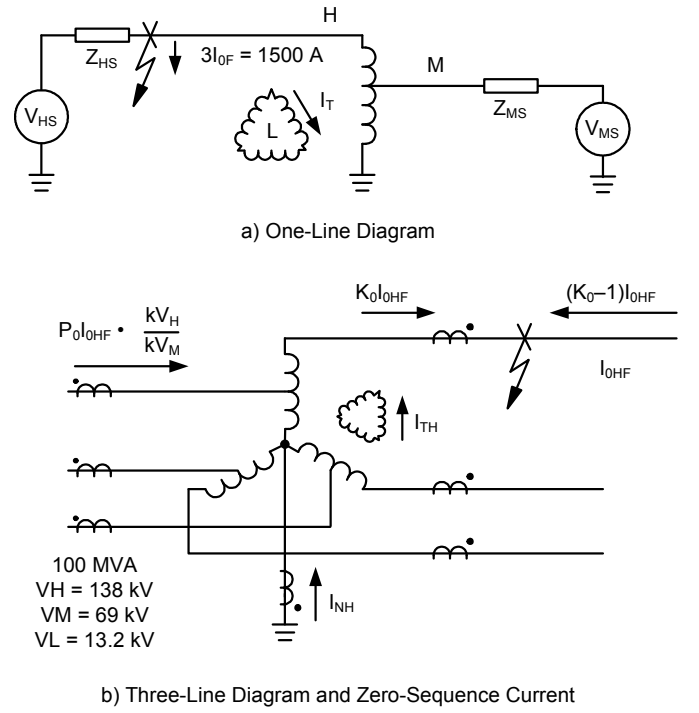b) Three-Line Diagram and Zero-Sequence Current

Fig. 5. Example System for Fault Calculation

$$I_N = 3 \cdot K_0 \cdot I_{0F} \cdot \left(1 - \frac{Z_L}{Z_L + Z_M + Z_{0MS}} \cdot \frac{kV_H}{kV_M}\right) \quad (1)$$

$$I_T = 3 \cdot K_0 \cdot I_{0F} \cdot \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}}\right)\frac{kV_H}{\sqrt{3} \cdot kV_L} \quad (2)$$

where

$$K_0 = \frac{Z_{0HS}}{Z_{0HZ} + Z_H + \left(\frac{Z_L(Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}\right)} = \frac{I_{0H}}{I_{0F}}$$

The equivalent zero-sequence network for the example system in Fig. 5 is shown in Fig. 6.
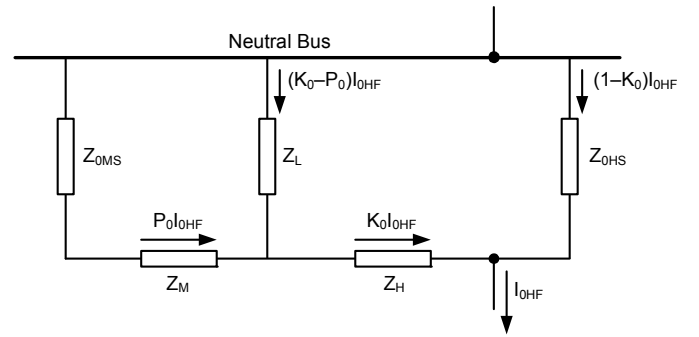

Fig. 6. Example System Zero-Sequence Network

A derivation of these equations is provided in the Appendix in Section VI. The current in the neutral and the current in the tertiary winding are different. Both are dependent on the system impedance and transformer impedances.

A simple example is used to illustrate how changing the system impedance can affect the magnitude and direction of the currents. Consider the system in Fig. 5 with the system and transformer impedances shown in Table I. The zero-sequence current in both the neutral and tertiary windings can be calculated using (1) and (2).

TABLE I
IMPEDANCE VALUES FOR CASE 1 FAULT CALCULATION

Circuit Component	Zero-Sequence Reactance (in Per Unit, 100 MVA Base)
ZL	0.1377
ZM	−0.0147
ZH	0.0870
Z0MS	0.1653
Z0HS	0.1034

Note that ZM is less than zero. This negative impedance is a result of the autotransformer mathematical representation as an equivalent wye set of impedances. Impedances for autotransformers are given typically as a percentage based on two windings connected in delta (e.g., ZML, ZHL, and ZHM). To perform fault calculations, we must first convert these impedances to an equivalent wye circuit consisting of three impedances, ZL, ZM, and ZH. This conversion often results in negative values for one of the three impedances, ZL, ZM, or ZH. This does not imply that the capacitance between two windings is being modeled, but is a result of the autotransformer mathematical representation. The fact that one of the impedances is negative makes the autotransformer circuit interesting, as we will see in the example calculation.

Assume there is a single-line-to-ground fault on A-phase with a fault current of 1500 amperes at the point of the fault. Substituting the values for impedances and total zero-sequence current into (1) and (2) yields:

$$K_0 = \frac{j0.1034}{j0.1034 + j0.0870 + \left(\frac{j0.1377(j0.1653 - j0.0147)}{j0.1377 + j0.1653 - j0.0147} \right)}$$

$$= 0.3942$$

$$I_N = 3 \cdot (0.3942) \cdot (500) \cdot \left(1 - \frac{j0.1377}{j0.1377 + j0.1653 - j0.0147} \cdot \frac{138}{69} \right)$$

$$= 26.46 \, A$$

$$I_T = 3 \cdot (0.3942) \cdot (500) \cdot \left(\frac{j0.1653 - j0.0147}{j0.1653 + j0.1377 - j0.0147} \right) \cdot \left(\frac{138}{\sqrt{3} \cdot 13.2} \right)$$

$$= 1864 \, A$$

Fig. 7 shows the current distribution throughout the equivalent zero-sequence network and the autotransformer terminals.
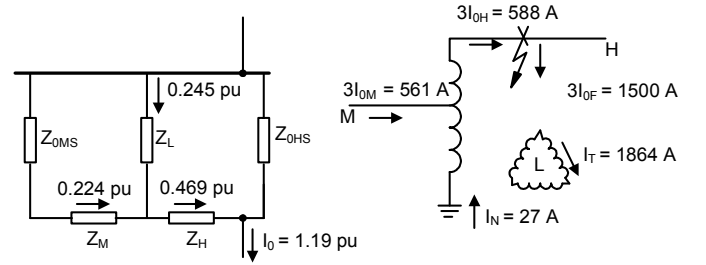

Fig. 7. Example System for Case 1 Fault Calculation

Consider the same fault on the same transformer with the same equivalent system impedance on the HV side. However, in this case, we use a lower equivalent system impedance on the autotransformer medium-voltage (MV) side. Using the impedance values in Table II, we can perform the same calculations to determine the current in the neutral and tertiary.

TABLE II
IMPEDANCE VALUES FOR CASE 2 FAULT CALCULATION

Circuit Component	Zero-Sequence Reactance (in Per Unit, 100 MVA Base)
ZL	0.1377
ZM	−0.0147
ZH	0.0870
Z0MS	0.0109
Z0HS	0.1034

$$K_0 = 0.5544$$

$$I_N = 3 \cdot (0.5544) \cdot (500) \cdot \left(1 - \frac{j0.1377}{j0.1377 + j0.0109 - j0.01476} \cdot \frac{138}{69} \right)$$

$$= -878 \, A$$

$$I_T = 3 \cdot (0.5544) \cdot (500) \cdot \left(\frac{j0.0109 - j0.0147}{j0.1377 + j0.0109 - j0.0147} \right) \cdot \left(\frac{138}{\sqrt{3} \cdot 13.2} \right)$$

$$= -142 \, A$$

Fig. 8 shows the current distribution throughout the equivalent zero-sequence network and the autotransformer terminals when the MV system impedance was lowered. Note that the zero-sequence currents in the neutral and tertiary have changed direction for a fault at the same location on the system. Also, the zero-sequence current on the MV side is larger (when expressed as per unit) than the current on the HV side.
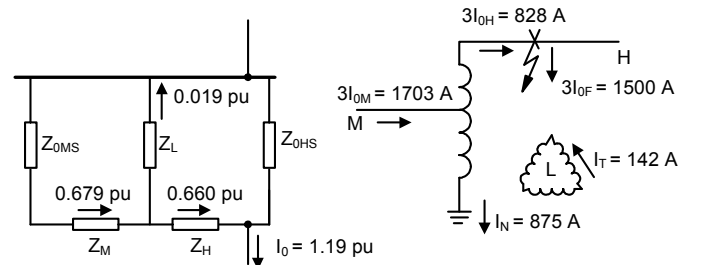

Fig. 8. Example System for Case 2 Fault Calculation

Even though both the neutral and tertiary currents changed direction, the relationship between their magnitudes is more complex than applying a simple scaling factor. In Case 2, the current in the neutral was almost 6 times larger than the current in the tertiary winding. In Case 1, the current in the tertiary was 69 times larger than the current in the neutral. When the system impedance was changed, both the neutral and tertiary currents changed direction for a fault in the same physical location. This scenario creates a problem for either current as a directional element polarizing quantity. Perform fault and system studies to determine the effect of realistic system impedance changes on neutral and tertiary current direction before using either current to polarize a ground directional element [3] [4] [5].

In autotransformer REF protection, the operate quantity Iop is not the residual current in one set of CTs but rather is the summation of the residual currents in the high-side and low-side windings. Which current, neutral or tertiary, is the correct quantity to use as a polarizing current for REF?

Fig. 9 gives a one-line representation of an autotransformer with a fault on the high side. The operating current and neutral polarizing current should be equal in magnitude and opposite in phase for an external fault. This is the case for both systems shown in Figs. 7 and 8, despite the fact that the neutral current direction changed. This is not the case if the tertiary currents shown in Figs. 7 and 8 are used as the polarizing current. From the equation for 32I (shown in Fig. 3), we can see that the cosine of 180 degrees is a negative value. A negative value for 32I indicates a reverse or out-of-zone fault.
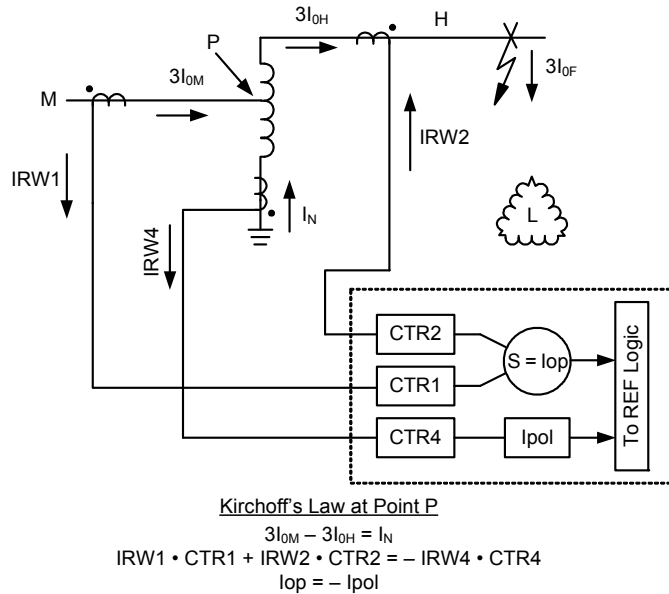

Kirchoff's Law at Point P
$$3I_{0M} - 3I_{0H} = I_N$$
$$IRW1 \cdot CTR1 + IRW2 \cdot CTR2 = -IRW4 \cdot CTR4$$
$$Iop = -Ipol$$

Fig. 9.   Relationship Between Iop and Ipol for Ideal External Fault on the Autotransformer HV Side

REF protection must use the neutral current as the polarizing quantity not the tertiary current. Equations (1) and (2) and the fault calculations summarized in Figs. 7 and 8 illustrate that the tertiary current and neutral current are different, and one cannot be substituted for another as a polarizing quantity for REF protection.

IV. REAL-WORLD CASE EXAMPLE

In March 2006, a REF element on a 100 MVA autotransformer operated unexpectedly. The customer queried the REF element operation. This section analyzes the unexpected operation and explains why the element operated.

The autotransformer includes a tertiary winding connected in delta. The voltage levels of the autotransformer are 138/69/13 kV. The MV winding supplies a dual 69 kV bus. Fig. 10 is a sketch of the autotransformer and REF element configuration.
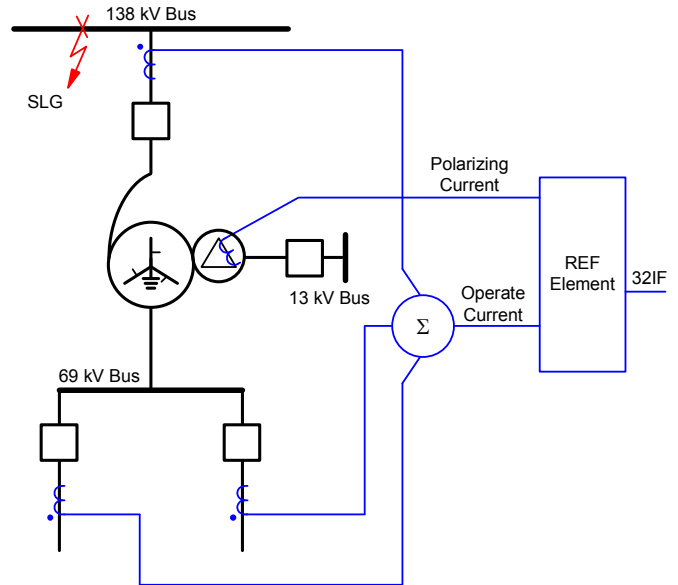

Fig. 10.   Autotransformer and REF Element Configuration

Fig. 11 is an oscillographic plot of the fault currents in secondary amperes. This plot shows the fault current distribution in the autotransformer during what, at first, appeared to be an external double-line-to-ground fault on the 138 kV transmission system. The double-line-to-ground fault assumption was made because of the phase currents observed in Fig. 11.
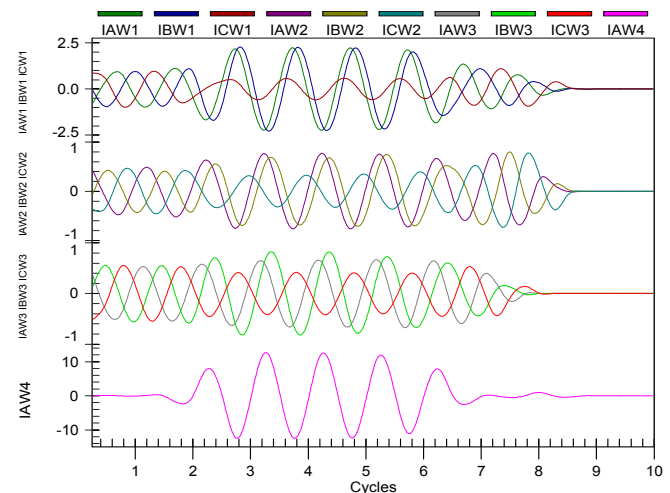

Fig. 11.   Autotransformer Fault Current Oscillographic Plot

Fig. 12 is an oscillographic plot of the currents and voltages in primary units of a nearby transmission line terminal that observed the same fault. A quick observation of Fig. 12 indicates that the fault was a B-phase-to-ground fault (BG).
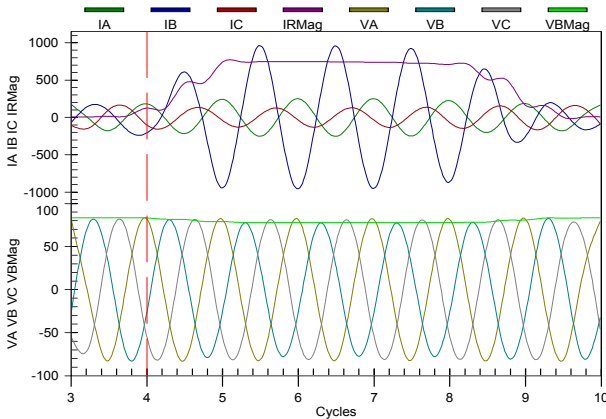

Fig. 12.   Oscillographic Plot of Fault Currents and Voltages for a BG Fault at a Nearby Transmission Line Terminal

Analyzing the sequence currents shown in Fig. 13 for the phase currents from the transformer relay in Fig. 11, we observe that the zero-sequence current phase angle leads the negative-sequence current by 120 degrees on all windings. Fault identification and selection (FIDS) logic found in commonly used microprocessor-based distance and directional relays indicated that the fault was either a BG fault or a CAG fault [6]. From this and the large B-phase fault current observed in Fig. 12, we conclude that this was a 138 kV BG fault.
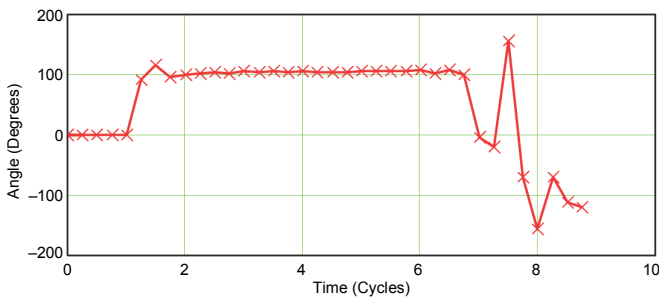

Fig. 13.   Angle Difference Between Zero-Sequence and Negative-Sequence Currents for Transformer Relay Winding One-Phase Currents in Fig. 11

Next, we analyze the transformer fault currents from Fig. 11 by calculating the sequence components in each of the windings. The positive- and negative-sequence components comply with the transformer transformation ratio, as expected. The sum of the MV (69 kV) winding zero-sequence currents is roughly equal to the current on the HV (138 kV) winding, and this does not abide by the transformation ratio. For all windings, however, the zero-sequence current angle leads the negative-sequence angle by roughly 120 degrees. This confirms that the fault type is BG.

The sum of the residual HV and MV winding currents make up the REF operate current. The tertiary winding (13 kV) current is used as the polarizing current. Using a Mathcad® file and data from the captured oscillographic data,

the operation of the REF element was simulated to understand why the element operated.

Fig. 14 is a plot of the residual (zero-sequence) currents present in the autotransformer HV and MV windings during the fault. The autotransformer MV winding residual current is simply the sum of the residual current in the two 69 kV breakers.
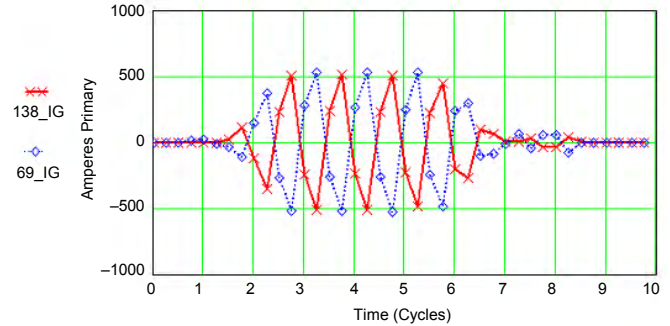

Fig. 14.   Residual Current in Autotransformer HV and MV Windings

Fig. 14 shows that the residual current in the HV winding is approximately equal in magnitude but of opposite phase to that in the MV winding. Fig. 15 is a plot of the sum of the HV and MV winding currents or the REF element operate current.
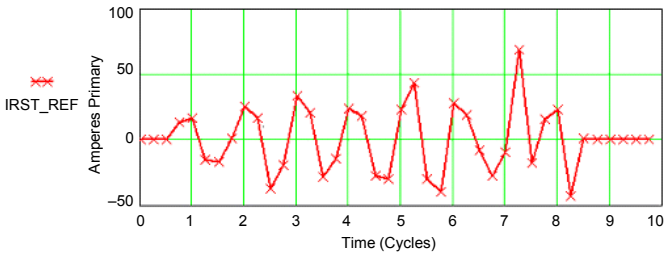

Fig. 15.   REF Element Operate Current Plot

In Fig. 15, the magnitude of the operate current in primary amperes is a fraction of the residual current in the HV or MV transformer windings.

Fig. 16 is a REF operate current plot obtained from the tertiary autotransformer winding.
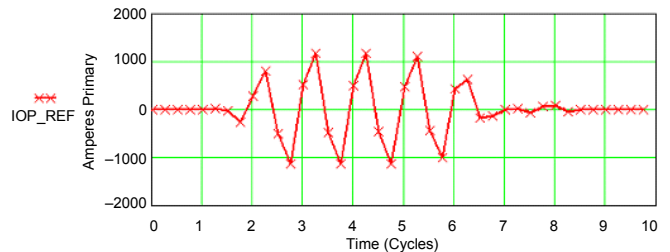

Fig. 16.   REF Element Polarizing Current Plot

Plotting the REF element operate and polarizing current on the same set of axes (Fig. 17) shows that the polarizing current is much larger than the operate current. Effectively, we can say that the operate current is nonexistent.
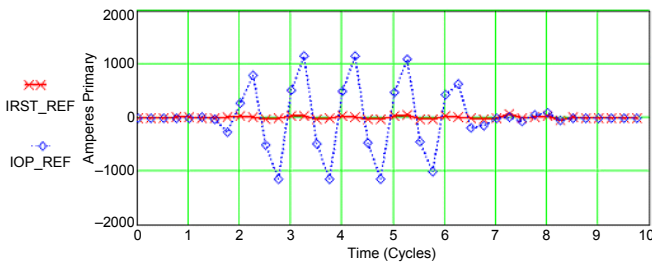
Fig. 17. REF Element Operate and Restraint Current Plot

Let us return our attention to the REF logic shown in Fig. 3. AND 1 was added to handle the situation shown in Fig. 18, whereby the grounded wye-connected winding of the transformer is disconnected from the power system, but the transformer is still connected to the power system via another winding (a delta-connected winding). If a ground fault were to develop on the wye-connected winding close to the neutral point, the phase differential element will not likely see the fault. No current would exist in the phase CTs, but current would exist in the transformer neutral. AND 1 (in Fig. 3) was added to protect a transformer under these conditions.
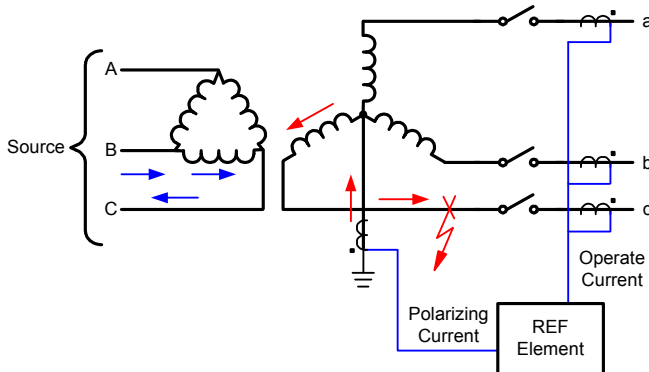

Fig. 18. Ground Fault on Wye Winding With Wye Line-End Open

Armed with the above information, look closely at the REF element operation and reexamine the REF logic in Figs. 2 and 3. For the torque calculation to be enabled, the restraint and operate current magnitudes must be above the pickup setting (50GP). Because the operate current is not above the pickup value, 50GC remains deasserted, and the torque calculation remains disabled. However, because the polarizing current is above the pickup threshold, AND 1 in Fig. 3 is enabled and the REF element determines that the fault is internal, having never checked the directional element decision. The REF element trips for the external BG fault.

So now we know why the REF element operated and our first instinct may be to blame the REF algorithm for being faulty. This is a good time to revisit the transformer theory discussed in Section III. For a normal delta-wye power transformer, if zero-sequence current flows in the delta winding, then zero-sequence current must flow in the wye-winding neutral. However, this is not the case for an autotransformer with a delta-connected tertiary winding. From (1) and (2) we see that it is possible to have current in the delta tertiary autotransformer winding and no current in the neutral. Unlike a normal transformer, the autotransformer HV and MV

windings are electrically and magnetically coupled. In a conventional delta-wye transformer, the windings are only magnetically coupled. This is the major difference between an autotransformer and a conventional transformer with respect to the zero-sequence path.

Fig. 14 shows that the zero-sequence currents in the autotransformer HV and MV windings had approximately the same magnitude and were 180 degrees out of phase. Overlaying this information on the one-line diagram in Fig. 19, and using Kirchhoff's current law, observe that almost no current flows in the autotransformer neutral. Stated another way, the zero-sequence current entering the MV winding exits the HV winding with no zero-sequence current contribution from the autotransformer neutral.
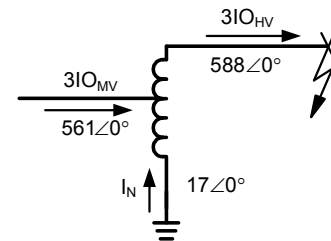

Fig. 19. Autotransformer One-Line With Zero-Sequence Fault Currents

This case clearly demonstrates that for an autotransformer, one should not use the current from the tertiary winding as the REF element polarizing current. Instead, use the current in the autotransformer neutral, as is the case in conventional transformers. Fig. 20 shows the recommended REF element connection for this specific case.
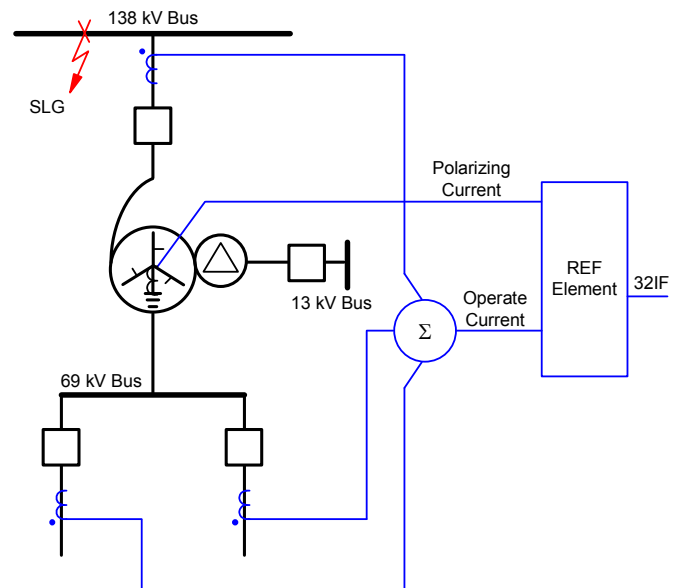

Fig. 20. Recommended Autotransformer REF Element Connection

If the autotransformer REF element would have been configured as shown in Fig. 20, the polarizing and operate current magnitude compared to the pickup threshold in primary amperes would have been as shown in Fig. 21b. Another interesting observation from Fig. 21a is that the operate and polarizing currents would have been of equal

magnitude and of opposite phase angles. The REF element would have remained secure for the out-of-zone fault [7].
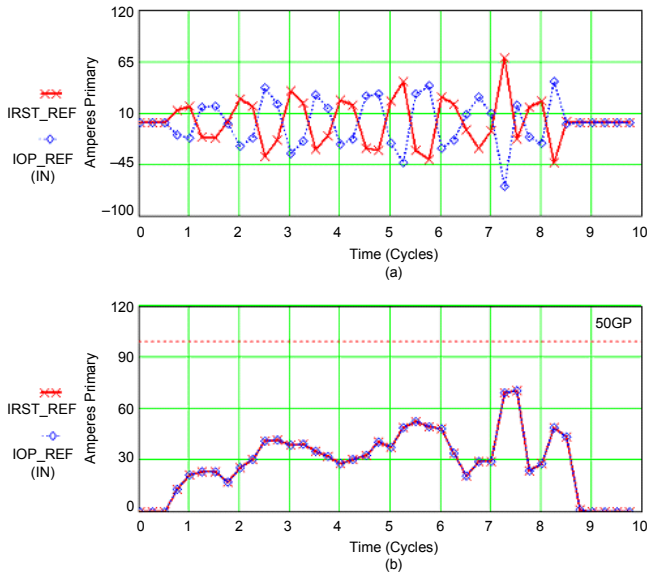

Fig. 21   Simulated Autotransformer REF Element Response With Recommended REF Element Connection

V. CONCLUSIONS

- REF elements compare zero-sequence current in the transformer neutral to zero-sequence current at the terminals of wye-connected windings to determine if a ground fault is internal to the transformer wye windings.
- REF sensitivity to ground faults is better than the phase differential element, especially at higher load currents.
- The angle between zero- and negative-sequence currents is a commonly used and reliable fault-type indicator.
- As with any differential or directional application, phase angle measurement stability and current direction predictability are critical to successful operation.
- For a normal delta-wye transformer, if zero-sequence current flows in the delta winding, then zero-sequence current flows in the wye winding neutral.
- In autotransformers with a delta-connected tertiary, it is possible to have zero-sequence current in the delta winding and none in the neutral because of magnetic and electrical coupling between the HV and MV windings.
- REF applications with autotransformers should use neutral currents rather than delta-tertiary currents for the operate quantity.
- Relay data and event analysis are key to a better understanding of power system operations and improving reliability.

VI. APPENDIX: AUTOTRANSFORMER NEUTRAL AND TERTIARY CURRENT DERIVATION EQUATION

A. Typical Datasheet

Fig. A.1.   Typical Autotransformer Transformer Datasheet

From the example transformer datasheet in Figure A.1, the impedance values are typically given from one winding to another, or what is referred to as the "network basis." The impedance values are given in per unit with the MVA base specified for each different impedance value.

B. Equivalent Wye Circuit

If the magnetizing impedance is neglected, the autotransformer can be represented by a set of three wye-connected impedances [4]. The process for determining these impedances is to select three pairs of terminals (e.g., H, M, and L), and calculate the equivalent wye impedances. Figs. A.2.a and A.2.b show the two system representations.
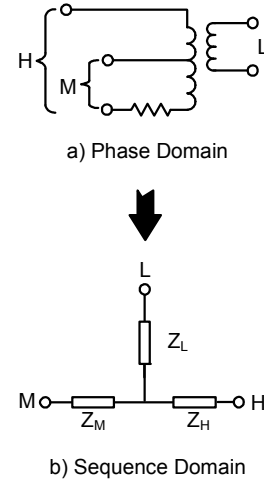

Fig. A.2.   a) One-Line Equivalent Phase Domain Circuit b) Sequence Domain Equivalent Circuit

Recognizing that Z_{HL} is the measured impedance when L is short-circuited and M is open-circuited and applying this information to Fig. A.2.b, we deduce the following relationships:

$$Z_{HL} = Z_H + Z_L \qquad (A.1)$$

$$Z_{HM} = Z_H + Z_M \qquad (A.2)$$

$$Z_{ML} = Z_M + Z_L \qquad (A.3)$$

These three equations form a linear system of equations, (three equations and three unknowns), which can be solved using a variety of methods, yielding:

$$Z_H = \frac{1}{2}\left(Z_{HL} + Z_{HM} - Z_{ML}\right) \qquad (A.4)$$

$$Z_M = \frac{1}{2}\left(Z_{HM} + Z_{ML} - Z_{HL}\right) \qquad (A.5)$$

$$Z_L = \frac{1}{2}\left(Z_{ML} + Z_{HL} - Z_{HM}\right) \qquad (A.6)$$

It is important to note that because a term is subtracted, often one of the three equivalent wye impedances is a negative value. This is a result of the mathematical representation of the circuit and should not be interpreted to represent a capacitance in the autotransformer [5].

C. Accounting for Grounding Impedance

A common practice in some systems is to ground the autotransformer through an impedance. This impedance will affect the zero-sequence representation of the autotransformer circuit. The equations for Z_H, Z_M, and Z_L above did not include this impedance.

The steps for deriving the equivalent circuit are shown in Fig. A.3.

Consider our original wye-equivalent circuit for the transformer shown in Fig. A.2.b; notice that there is no neutral conductor shown. How do you modify the model to account for the additional neutral impedance? Where does the impedance get added? We must consider a different autotransformer model.

Consider Fig. A.3.b, and define the following impedances:

$$Z_{S,TOT} = Z_S$$
$$Z_{C,TOT} = Z_C + 3 \cdot Z_{nH}$$
$$Z_{T,TOT} = Z_T$$

In other words, the impedance seen at the S terminals is the leakage reactance of Winding S. The same is true for the T terminals. However, for the C terminals, the impedance consists of not only the leakage reactance of Winding C but also the neutral impedance. Representing the autotransformer in terms of three different sets of Terminals S, C, and T is known as the "winding basis" and may be more convenient to use when designing the transformer itself. However, representing the autotransformer using a winding basis does not translate well into fault calculations and system studies. Armed with this information, compute Z_{HL}, Z_{ML}, and Z_{HM} using the circuit parameters Z_S, Z_T, and Z_C.
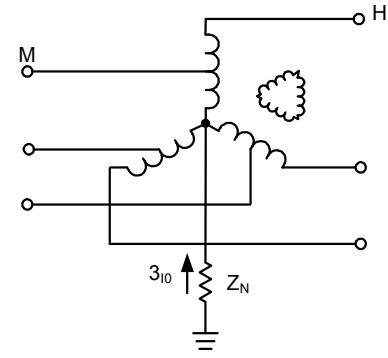
a) Phase Domain Representation

$$Z_{nH} = \frac{V_H}{V_M}$$
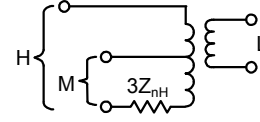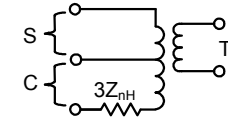
b) Winding or Network Basis Representation
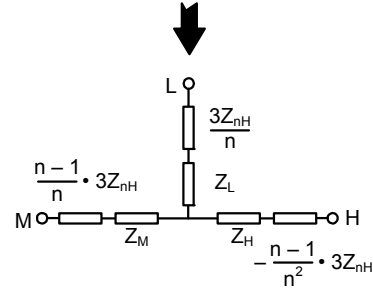
c) Equivalent Zero-Sequence Network

d) Simplified Equivalent Zero-Sequence Network

$$\begin{bmatrix} Z_L^{'} \\ Z_M^{'} \\ Z_H^{'} \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 & -1 & \frac{1}{n} \\ 1 & -1 & 1 & \frac{n-1}{n} \\ -1 & 1 & 1 & -\frac{n-1}{n^2} \end{bmatrix} \cdot \begin{bmatrix} Z_{ML} \\ Z_{HL} \\ Z_{HM} \\ 6Z_{nH} \end{bmatrix}$$

Fig. A.3.   Derivation of Sequence-Domain Equivalent Circuit With Neutral Grounding Impedance Included

We now recall that Z_{HL} was the measured impedance when a voltage (V_{test}) is applied to the H terminals with the L terminals short-circuited and the M terminals open-circuited, as shown in Fig. A.4.

Fig. A.4. Test for Measuring Z_{HL}

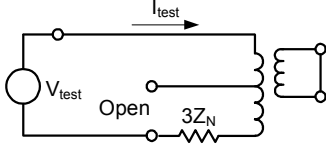It is important to note that Z_{HM}, Z_{HL}, and Z_{ML} are expressed in per unit, whereas our circuit above is expressed in terms of volts and amperes. We will account for this by performing our calculation in a common base. We will also assume that Z_C, Z_S, and Z_T are given as equivalent ohms or ohms at the common base. This process is very similar to per unit. It involves selecting a common MVA base just as per unit calculations do; however, we will also select one voltage base and convert all impedances to that voltage base regardless of which transformer winding they are on. In order for our "common base" model to avoid violating the laws of physics, we have to account for the turns ratio, just as if we had left the quantities in terms of actual ohms. So we can define:

$$n = \frac{V_H}{V_M} = \frac{V_S + V_C}{V_C}$$

So:

$$\frac{V_S}{V_C} = n - 1$$

V_{test} is then calculated as:

$$V_{test} = n^2 \cdot Z_T \cdot I_{test} + (n-1)^2 \cdot Z_S \cdot I_{test} + (Z_C + 3 \cdot Z_N) \cdot I_{test} \quad (A.7)$$

We can calculate $Z_{HL,act}$:

$$Z_{HL,act} = \frac{V_{test}}{I_{test}} = n^2 \cdot Z_T + (n-1)^2 \cdot Z_S + (Z_C + 3 \cdot Z_N) \quad (A.8)$$

If we choose the MV Winding M (or C) as our voltage base and consider the transformer rating as a common MVA base for all of our quantities, we can then convert Z_{HL} from actual ohms to equivalent ohms or ohms on voltage base M:

$$Z_{HL,baseC} = \frac{Z_{HL,act}}{n^2} = Z_T + \left(\frac{n-1}{n}\right)^2 \cdot Z_S + \left(\frac{Z_C + 3 \cdot Z_N}{n^2}\right) \quad (A.9)$$

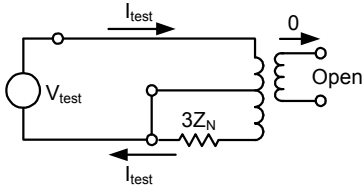Similarly, we can perform the same calculation for Z_{HL}.

Fig. A.5. Test for Measuring Z_{HM}

$$V_{test} = (n-1)^2 \cdot (Z_S + Z_C + 3 \cdot Z_N) \cdot I_{test} \quad (A.10)$$

$$Z_{HM,act} = \frac{V_{test}}{I_{test}} = (n-1)^2 \cdot (Z_S + Z_C + 3 \cdot Z_N) \quad (A.11)$$

$$Z_{HM,baseC} = \frac{Z_{HM,act}}{n^2} = \left(\frac{n-1}{n}\right)^2 \cdot (Z_S + Z_C + 3 \cdot Z_N) \quad (A.12)$$

Finally, it should be evident that:

$$Z_{ML,baseC} = Z_C + Z_T \quad (A.13)$$

We now have calculated the impedances $Z_{HL,baseC}$, $Z_{HM,baseC}$, and $Z_{ML,baseC}$ in equivalent ohms using a voltage base of V_C. However, a typical nameplate gives these values in terms of per unit, not in ohms or equivalent ohms. So we can convert by simply taking each impedance in equivalent ohms and dividing by the $Z_{base,C}$, yielding:

$$Z_{HL} = \frac{Z_{HL,baseC}}{Z_{baseC}} = \frac{Z_T}{Z_{baseC}} + \left(\frac{n-1}{n}\right)^2 \cdot \frac{Z_S}{Z_{baseC}} + \left(\frac{\frac{Z_C}{Z_{baseC}} + 3 \cdot \frac{Z_N}{Z_{baseC}}}{n^2}\right) \quad (A.14)$$

$$= Z_{T,pu} + \left(\frac{n-1}{n}\right)^2 \cdot Z_{S,pu} + \left(\frac{Z_{C,pu} + 3 \cdot Z_{nH}}{n^2}\right)$$

$$Z_{HM} = \frac{Z_{HM,baseC}}{Z_{baseC}} = \left(\frac{n-1}{n}\right)^2 \cdot \left(\frac{Z_S + Z_C + 3 \cdot Z_N}{Z_{baseC}}\right)$$

$$= \left(\frac{n-1}{n}\right)^2 \cdot (Z_{S,pu} + Z_{C,pu} + 3 \cdot Z_{nH}) \quad (A.15)$$

$$Z_{ML} = \frac{Z_{ML,baseC}}{Z_{baseC}} = \frac{Z_C + Z_T}{Z_{baseC}}$$

$$= Z_{C,pu} + Z_{T,pu} \quad (A.16)$$

where:

$$Z_{nH} = \frac{Z_N}{Z_{baseC}}$$

We can take (A.14), (A.15), and (A.16) and substitute into (A.4), (A.5), and (A.6) to calculate Z_H, Z_M, and Z_L.

$$Z_H' = \left(\frac{n-1}{n}\right)^2 \cdot Z_{S,pu} - \frac{n-1}{n^2} \cdot (Z_{C,pu}) - \frac{n-1}{n^2} 3 \cdot Z_{nH}$$

$$= Z_H - \frac{n-1}{n^2} 3 \cdot Z_{nH} \quad (A.17)$$

$$Z_M' = \frac{n-1}{n} \cdot (Z_{C,pu}) + \frac{n-1}{n} 3 \cdot Z_{nH}$$

$$= Z_M + \frac{n-1}{n} 3 \cdot Z_{nH} \quad (A.18)$$

$$Z_L' = Z_{T,pu} + \frac{1}{n} \cdot (Z_{C,pu}) + \frac{1}{n} 3 \cdot Z_{nH}$$

$$= Z_L + \frac{1}{n} 3 \cdot Z_{nH} \quad (A.19)$$

Notice that (A.17), (A.18), and (A.19) match the equivalent wye network shown in Figure A.3.c.

Alternatively, we can write (A.17), (A.18), (A.19) in a matrix form:

$$\begin{bmatrix} Z_L' \\ Z_M' \\ Z_H' \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 & -1 & \frac{1}{n} \\ 1 & -1 & 1 & \frac{n-1}{n} \\ -1 & 1 & 1 & -\frac{n-1}{n^2} \end{bmatrix} \cdot \begin{bmatrix} Z_{ML} \\ Z_{HL} \\ Z_{HM} \\ 6Z_{nH} \end{bmatrix} \quad (A.20)$$

D. Zero-Sequence Network Representation

Using the wye-equivalent circuit derived, we can draw an equivalent-sequence network. To make our equivalent-sequence networks general in nature, use an equivalent source for both systems on the MV and HV side.

Because our derivation is focused on zero-sequence current in the neutral and tertiary, we are concerned with zero-sequence current and only the zero-sequence network is drawn.

The network can then be reduced and the total sequence currents, positive, negative, and zero sequence, can be calculated by connecting the sequence networks appropriately for the fault type. After calculating the total positive-, negative-, and zero-sequence currents at the point of fault, we examine the contribution from each source or branch within the circuit, and this is the basis for deriving the current in the neutral and tertiary branches.

a) Low-Side Fault
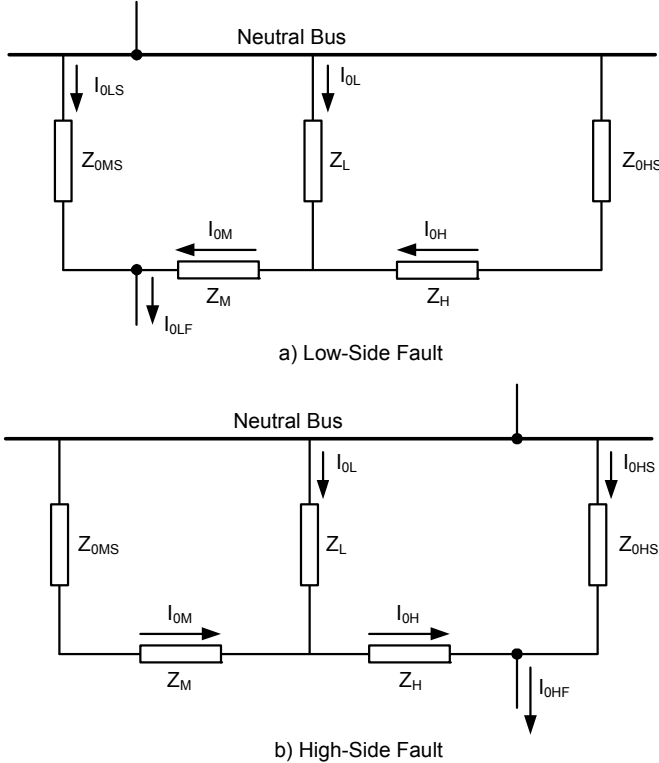

b) High-Side Fault

Fig. A.6. Equivalent Zero-Sequence Network for a) A Ground Fault on the Autotransformer Low Side b) A Ground Fault on the Autotransformer High Side

Considering the external fault on the autotransformer high side with the equivalent zero-sequence network shown in Fig. A.6.b, we can use a current divider to calculate the zero-sequence current on the autotransformer high side as follows:

$$I_{0Hpu} = \frac{Z_{0HS}}{Z_{0HS} + Z_H + \frac{Z_L \cdot (Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}} \cdot I_{0HFpu} \quad (A.21)$$

Similarly, we can apply a current divider to obtain the current in the tertiary winding

$$I_{0Lpu} = \frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \cdot I_{0Hpu}$$

$$= \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot \left(\frac{Z_{0HS}}{Z_{0HS} + Z_H + \frac{Z_L \cdot (Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}} \right) \cdot I_{0HFpu} \quad (A.22)$$

and on the MV side of the transformer.

$$I_{0Mpu} = \frac{Z_L}{Z_L + Z_M + Z_{0MS}} \cdot I_{0Hpu}$$

$$= \left(\frac{Z_L}{Z_L + Z_M + Z_{0MS}} \right) \cdot \left(\frac{Z_{0HS}}{Z_{0HS} + Z_H + \frac{Z_L \cdot (Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}} \right) \cdot I_{0HFpu} \quad (A.23)$$

In order to simplify the expressions, we can define current distribution factors such that:

$$I_{0Hpu} = K_0 \cdot I_{0HFpu} \quad (A.24)$$

and

$$I_{0Mpu} = P_0 \cdot I_{0HFpu} \quad (A.25)$$

If we then look at the sequence diagram in Fig. A.6.b and apply Kirchoff's law, then:

$$I_{0Lpu} = (K_0 - P_0) \cdot I_{0HFpu} \quad (A.26)$$

Comparing (A.24) to (A.21) we can see that:

$$K_0 = \frac{Z_{0HS}}{Z_{0HS} + Z_H + \frac{Z_L \cdot (Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}} \quad (A.27)$$

Then, by comparing (A.26) to (A.22):

$$K_0 - P_0 = \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot \left(\frac{Z_{0HS}}{Z_{0HS} + Z_H + \frac{Z_L \cdot (Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}} \right) \quad (A.28)$$

Substituting for K_0:

$$K_0 - P_0 = \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \quad (A.29)$$

Finally, comparing (A.25) to (A.23):

$$P_0 = \left(\frac{Z_L}{Z_L + Z_M + Z_{0MS}} \right) \cdot \left(\frac{Z_{0HS}}{Z_{0HS} + Z_H + \frac{Z_L \cdot (Z_M + Z_{0MS})}{Z_L + Z_M + Z_{0MS}}} \right) \quad (A.30)$$

Substituting,

$$P_0 = \left(\frac{Z_L}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \quad (A.31)$$

We can then express the tertiary current as:

$$I_{0Lpu} = \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \cdot I_{0HFpu} \quad \text{(A.32)}$$

This is given in per unit. It may be useful to consider the values in actual amperes. If we use the total zero-sequence current in amperes (I_{0HF}) rather than in per unit (I_{0Hfpu}) and substitute into (A.32) we get:

$$I_{0L} = \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \cdot I_{0HF} \cdot \frac{I_{base,LOW}}{I_{base,HIGH}} \quad \text{(A.33)}$$

but

$$\frac{I_{base,LOW}}{I_{base,HIGH}} = \frac{kV_H}{kV_L} \quad \text{(A.34)}$$

Now consider that the zero-sequence current in the tertiary is "trapped" inside the delta. The base quantities for per-unit calculation are line currents not delta currents. Taking this into consideration and substituting into (A.33) we arrive at:

$$I_{0L} = \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \cdot I_{0HF} \cdot \frac{kV_H}{kV_L \cdot \sqrt{3}} \quad \text{(A.35)}$$

Because the neutral current does not appear on the zero-sequence network diagram, we consider Fig. A.7 and apply Kirchoff's Law to obtain the neutral current from 3_{I0H} and 3_{I0M}. Again, it is important to note that we are now dealing in the phase domain and the values are in amperes not per unit.
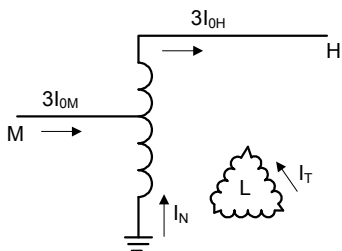

Fig. A.7. Kirchoff's Law Applied to Autotransformer to Derive I_N

Applying Kirchoff's Law yields:

$$I_N = 3I_{0H} - 3I_{0M} \quad \text{(A.36)}$$

We have expressions for I_{0Hpu} and I_{0Mpu} given in (A.24) and (A.25); however, they are in per unit and not amperes.

Converting to amperes and substituting into (A.36) yields:

$$I_N = 3 \cdot K_0 I_{0HF} - 3 \cdot P_0 \cdot I_{0HF} \cdot \frac{kV_H}{kV_M} \quad \text{(A.37)}$$

Substituting for P_0 from (A.31):

$$I_N = 3 \cdot K_0 I_{0HF} - 3 \cdot \left(\frac{Z_L}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \cdot I_{0HF} \cdot \frac{kV_H}{kV_M} \quad \text{(A.38)}$$

Finally, we can factor out the common terms and are left with:

$$I_N = 3 \cdot K_0 I_{0HF} \left(1 - \left(\frac{Z_L}{Z_L + Z_M + Z_{0MS}} \right) \cdot \frac{kV_H}{kV_M} \right) \quad \text{(A.39)}$$

Thus the current in the neutral and tertiary for the high-side fault are given in (A.39) and (A.35), respectively.

For brevity, the current in the neutral and tertiary windings for the low-side fault are not included; however, the process is similar. The equations for both cases are summarized in Table A-I.

Fig. A.8 shows the zero-sequence diagrams for both cases with the current distribution factors included.

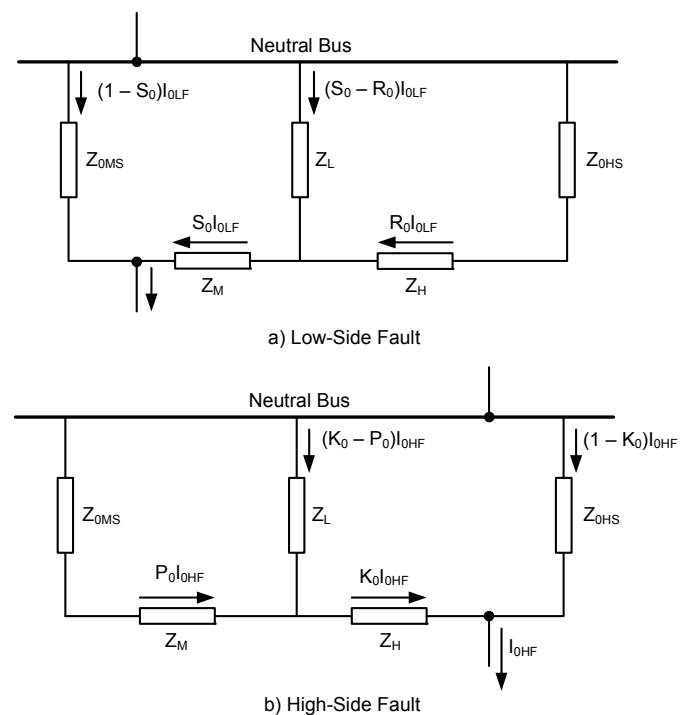a) Low-Side Fault

b) High-Side Fault

Fig. A.8. Zero-Sequence Networks With Current Distribution Factors for a) A Ground Fault on the Autotransformer High Side and b) A Ground Fault on the Fault Low Side

TABLE A-I:
NEUTRAL AND TERTIARY CURRENTS FOR BOTH FAULTS ON THE AUTOTRANSFORMER HIGH SIDE AND LOW SIDE

Fault Location	Neutral Current (Amperes)	Tertiary Current (Amperes)
Autotransformer High Side	$I_N = 3 \cdot K_0 I_{0HF} \left(1 - \left(\frac{Z_L}{Z_L + Z_M + Z_{0MS}} \right) \cdot \frac{kV_H}{kV_M} \right)$	$I_{0L} = \left(\frac{Z_M + Z_{0MS}}{Z_L + Z_M + Z_{0MS}} \right) \cdot K_0 \cdot I_{0HF} \cdot \frac{kV_H}{kV_L \cdot \sqrt{3}}$
Autotransformer Low Side	$I_N = 3 \cdot S_0 I_{0LF} \left(1 - \left(\frac{Z_L}{Z_L + Z_H + Z_{0HS}} \right) \cdot \frac{kV_M}{kV_H} \right)$	$I_{0L} = \left(\frac{Z_H + Z_{0HS}}{Z_L + Z_H + Z_{0HS}} \right) \cdot S_0 \cdot I_{0LF} \cdot \frac{kV_M}{kV_L \cdot \sqrt{3}}$

VII. ACKNOWLEDGMENTS

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IX. BIOGRAPHIES

Normann Fischer received a Higher Diploma in Technology, with honors, from Witwatersrand Technikon, Johannesburg in 1988, a BSc in Electrical Engineering, with honors, from the University of Cape Town in 1993, and an MSEE from the University of Idaho in 2005. He joined Eskom as a protection technician in 1984 and was a senior design engineer in Eskom's Protection Design Department for three years. He then joined IST Energy as a senior design engineer in 1996. In 1999, he joined Schweitzer Engineering Laboratories, Inc. as a power engineer in the Research and Development Division. He was a registered professional engineer in South Africa and a member of the South Africa Institute of Electrical Engineers.

Derrick Haas graduated from Texas A&M University in 2002 with a BSEE. He worked as a distribution engineer at CenterPoint Energy in Houston, Texas from 2002 to 2006. In April 2006, Derrick joined Schweitzer Engineering Laboratories, Inc. and works as a field application engineer in Houston, Texas.

David Costello graduated from Texas A&M University in 1991 with a BSEE. He worked as a system protection engineer at Central Power and Light and Central and Southwest Services in Texas and Oklahoma. He has served on the System Protection Task Force for ERCOT. In 1996, David joined Schweitzer Engineering Laboratories, Inc. where he has served as a field application engineer and regional service manager. He presently holds the title of senior application engineer and works in Boerne, Texas. He is a senior member of IEEE and a member of the planning committee for the Conference for Protective Relay Engineers at Texas A&M University.