Tutorial on Symmetrical Components

Part 2: Answer Key

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***Abstract*—Symmetrical components and the per-unit system are two of the most fundamental and necessary types of mathematics for relay engineers and technicians. We must practice these techniques in order to fully understand and feel comfortable with them. This white paper provides both theoretical and real-world examples with questions and solutions that can be used to gain experience with symmetrical components.**

I. Introduction

The method of symmetrical components is used to simplify fault analysis by converting a three-phase unbalanced system into two sets of balanced phasors and a set of single-phase phasors, or symmetrical components. These sets of phasors are called the positive-, negative-, and zero-sequence components. These components allow for the simple analysis of power systems under faulted or other unbalanced conditions. Once the system is solved in the symmetrical component domain, the results can be transformed back to the phase domain.

The topic of symmetrical components is very broad and can take considerable time to cover in depth. A summary of important points is included in this introduction, although it is highly recommended that other references be studied for a more thorough explanation of the mathematics involved. Refer to [1], [2], [3], [4], and [5] for more information on symmetrical components.

A. Converting Between the Phase and Symmetrical Component Domains

Any set of phase quantities can be converted into symmetrical components, where α is defined as $1\angle 120$, as follows:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (1)$$

where I_0, I_1, and I_2 are the zero-, positive-, and negative-sequence components, respectively. This equation shows the symmetrical component transformation in terms of currents, but the same equations are valid for voltages as well.

This results in the following equations:

$$\begin{aligned} I_0 &= \frac{1}{3}(I_A + I_B + I_C) \\ I_1 &= \frac{1}{3}(I_A + \alpha I_B + \alpha^2 I_C) \\ I_2 &= \frac{1}{3}(I_A + \alpha^2 I_B + \alpha I_C) \end{aligned} \quad (2)$$

Likewise, a set of symmetrical components can be converted into phase quantities as follows:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad (3)$$

This results in the following equations:

$$\begin{aligned} I_A &= I_0 + I_1 + I_2 \\ I_B &= I_0 + \alpha^2 I_1 + \alpha I_2 \\ I_C &= I_0 + \alpha I_1 + \alpha^2 I_2 \end{aligned} \quad (4)$$

These conversions are valid for an A-phase base, which can be used for A-phase-to-ground, B-phase-to-C-phase, B-phase-to-C-phase-to-ground, and three-phase faults. In Section V, Example 4 shows how the base changes for other irregular fault types. These conversions are also only valid for an ABC system phase rotation. In Section VI, Example 5 shows how the equations change for an ACB system phase rotation.

A calculator was created in Microsoft® Excel® to allow us to convert between the phase and symmetrical component domains. This calculator is available for download with this white paper at http://www.selinc.com/.

B. Transformer Representations in the Sequence Networks

For information on the formation of the sequence networks as well as the representation of power system components in the sequence networks, see [1] and [2].

Transformers are simply represented as their positive- and negative-sequence impedances in the positive- and negative-sequence networks, respectively. However, the transformer representation in the zero-sequence network can be more complex and is dependent on the type of transformer connection. Fig. 1 shows some common transformer connections and the equivalent zero-sequence representations. For a complete list of transformer connections, see [1].

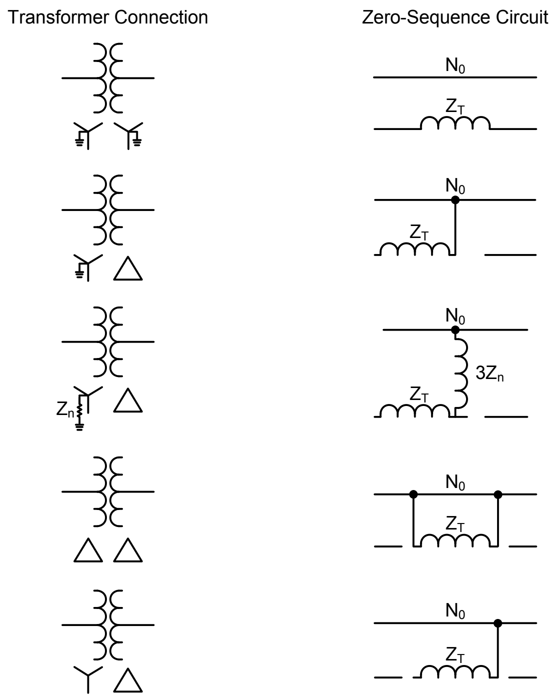


Fig. 1. Zero-sequence circuits for various transformer types

C. Connecting the Sequence Networks

Once the sequence networks for the system are defined, the way they are connected is dependent on the type of fault. Sequence network connections for common shunt fault types are shown in the remainder of this subsection. For complete derivations of these network connections as well as sequence network connections for series faults, see [2]. In the connections that follow, ZF is defined as the fault impedance from each phase to the common point, and ZG is defined as the impedance from the common point to ground. The ZG term is only significant when ZF differs per phase or if the line impedance to the fault point is different between phases. The typical assumptions are that ZF is the same across all phases and the line impedances are equal, and therefore, the ZG term is neglected.

For a three-phase fault, the positive-sequence network is used with the fault point connected back to the neutral bus, as shown in Fig. 2.

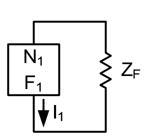


Fig. 2. Sequence network connections for a three-phase fault

For a single-phase-to-ground fault, the three networks are connected in series. Any fault impedance is multiplied by 3 and included in this connection, as shown in Fig. 3.

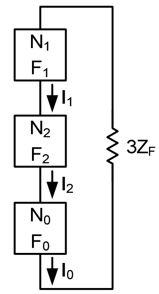


Fig. 3. Sequence network connections for a single-phase-to-ground fault

For a phase-to-phase fault, the positive- and negative-sequence networks are connected in parallel, as shown in Fig. 4.

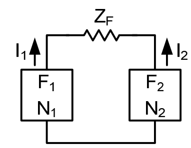


Fig. 4. Sequence network connections for a phase-to-phase fault

For a double-line-to-ground fault, all three networks are connected in parallel, as shown in Fig. 5.

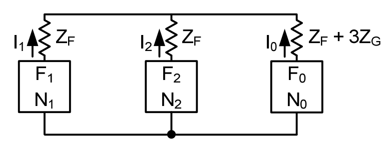


Fig. 5. Sequence network connections for a double-line-to-ground fault

D. The Per-Unit System

The per-unit system puts all the values of a power system on a common base so they can be easily compared across the entire system. To use the per-unit system, we normally begin by selecting a three-phase power base and a line-to-line voltage base. We can then calculate the current and impedance bases using the chosen power and voltage bases as shown:

$$I_{base} = \frac{S_{base}}{\sqrt{3} \bullet V_{base}} \tag{5}$$

$$Z_{base} = \frac{\left(V_{base}\right)^2}{S_{base}} \tag{6}$$

Any power system value can be converted to per unit by dividing the value by the base of the value, as shown:

$$\text{Quantity in per unit} = \frac{\text{Actual quantity}}{\text{Base value of quantity}} \tag{7}$$

Likewise, a per-unit value can be converted to an actual quantity at any time by multiplying the per-unit value by the base value of that quantity.

To convert impedances from one base to another, use the following equation:

$$Z_{pu}^{new} = Z_{pu}^{old} \bullet \frac{S_{base}^{new}}{S_{base}^{old}} \left(\frac{V_{base}^{old}}{V_{base}^{new}} \right)^2 \tag{8}$$

For more information on the per-unit system, see [1].

E. Examples

The rest of this paper consists of theoretical and practical examples that can be used to practice and gain experience in symmetrical component and per-unit techniques. Each example consists of questions to guide the reader through the analysis as well as complete solutions. In the cases with real-world events, the event records from the relays are available for download with this white paper and the reader should use ACSELERATOR Analytic Assistant® SEL-5601 Software to view them (available for free download at http://www.selinc.com).

II. Example 1: Single-Phase Versus Three-Phase Fault Current

This example shows how to calculate fault currents for two different fault types at two different locations on a distribution system. Fig. 6 shows the radial system with two possible fault locations.

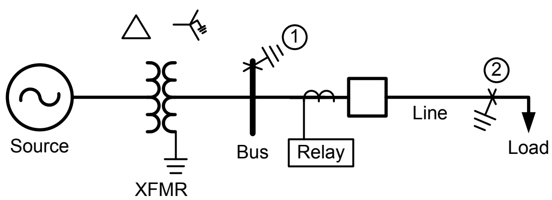


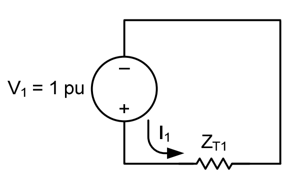
Fig. 6. Radial system with two fault locations

II-a On a radial distribution feeder, what type of fault do we expect to produce the largest fault current?

This depends on the fault location and transformer type, as we see in this example.

II-b Using symmetrical components, solve for the maximum fault current for a bolted three-phase fault at Location 1.

Because a three-phase fault is balanced, no negative- or zero-sequence currents are present, and therefore, only the positive-sequence network is used. The following figure shows the positive-sequence network with only the positive-sequence impedance of the transformer, because the fault is just past the secondary windings of the transformer.



The positive-sequence current can be solved for by dividing the positive-sequence voltage by the positive-sequence impedance of the transformer.

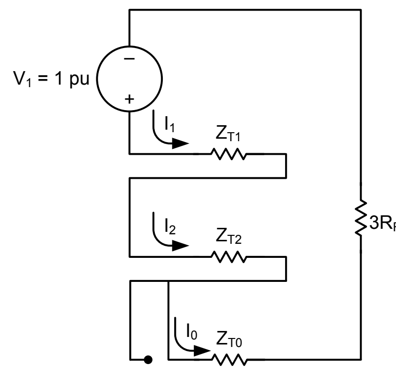
$$I_1 = \frac{V_1}{Z_{T1}}$$

Because $I_A = I_0 + I_1 + I_2$ and I_0 and I_2 are zero, then:

$$I_A = I_1 = \frac{1}{Z_{T1}}$$

II-c Using symmetrical components, solve for the maximum fault current for a phase-to-ground fault at Location 1.

The following figure shows the sequence networks connected in series for a single-phase-to-ground fault.



The positive-, negative-, and zero-sequence currents are equivalent and can be solved for by dividing the positive-sequence voltage by the equivalent impedance of the network.

$$I_1 = \frac{V_1}{Z_{T1} + Z_{T2} + Z_{T0} + 3R_F}$$

If we assume that $Z_{T1} = Z_{T2} = Z_{T0}$ and there is zero fault resistance, then:

$$I_1 = \frac{1}{3Z_{T1}} = I_2 = I_0$$

$$I_A = I_0 + I_1 + I_2 = \frac{1}{Z_{T1}}$$

II-d Assume a core-type transformer with a zero-sequence impedance of 85 percent of the positive-sequence impedance. Solve for the fault current for a phase-to-ground fault at Location 1, and compare the results with that of a three-phase fault.

A core-type transformer has a lower exciting impedance, and the zero-sequence impedance can be 85 to 100 percent of the positive-sequence impedance [6]. If we assume a core-type transformer, then $Z_{T0} = 0.85 \bullet Z_{T1}$.

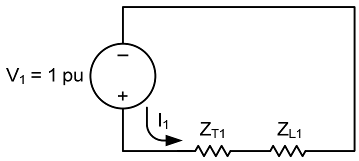
$$I_1 = \frac{V_1}{Z_{T1} + Z_{T2} + 0.85 \bullet Z_{T1}} = \frac{1}{2.85 \bullet Z_{T1}} = I_2 = I_0$$

$$I_A = I_0 + I_1 + I_2 = \frac{3}{2.85 \bullet Z_{T1}} = \frac{1.05}{Z_{T1}}$$

Assuming a core-type transformer, a phase-to-ground fault can produce more fault current than a three-phase fault when the fault is at the bus. The event report titled **Example 1A.cev** shows an evolving fault where the fault current for the line-to-ground fault is larger than that of the three-phase fault. See [7] and [8] for a complete analysis of this event.

II-e Using symmetrical components, solve for the maximum fault current for a three-phase fault at Location 2.

The sequence network for the new fault location is the same as for the previous fault location, except now we have the line impedance included. This is shown in the following figure.



The positive-sequence current can be solved for as follows:

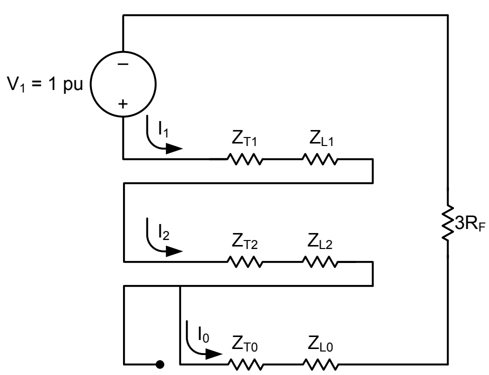
$$I_1 = \frac{V_1}{Z_{T1} + Z_{L1}}$$

Because $I_A = I_0 + I_1 + I_2$ and I_0 and I_2 are zero, then:

$$I_A = I_1 = \frac{1}{Z_{T1} + Z_{L1}}$$

II-f Using symmetrical components, solve for the maximum fault current for a phase-to-ground fault at Location 2. Is this greater than or less than the fault current for a three-phase fault?

The following figure shows the sequence networks connected in series for a single-phase-to-ground fault, with the line impedance included because of the new fault location.



The positive-, negative-, and zero-sequence currents are equivalent and can be solved for by dividing the positive-sequence voltage by the equivalent impedance of the network.

$$I_1 = \frac{V_1}{Z_{T1} + Z_{L1} + Z_{T2} + Z_{L2} + Z_{T0} + Z_{L0} + 3R_F}$$

Assume that $Z_{T1} = Z_{T2} = Z_{T0}$ and there is zero fault resistance. Also assume that $Z_{L1} = Z_{L2}$ and $Z_{L0} = 3 \bullet Z_{L1}$.

$$I_1 = \frac{1}{3Z_{T1} + 5Z_{L1}} = I_2 = I_0$$

$$I_A = I_0 + I_1 + I_2 = \frac{3}{3Z_{T1} + 5Z_{L1}} = \frac{1}{Z_{T1} + (1.67 \bullet Z_{L1})}$$

Comparing the results for the fault at Location 2, we can conclude that for a fault out on the feeder, the fault current produced by a three-phase fault is larger than that produced by a single-phase-to-ground fault. This is because, for a fault out on the feeder, the zero-sequence line impedance (which is typically larger than the positive-sequence line impedance) begins to dominate and make the line-to-ground fault current less than that of a three-phase fault. The event report titled **Example 1B.cev** shows an evolving fault where the fault current for the three-phase fault is larger than that of the line-to-ground fault. See [7] and [8] for a complete analysis of this event.

III. Example 2: Per-Unit System and Fault Calculations

This example shows how to work in the per-unit system and calculate fault currents for faults at the high-voltage terminals of the step-up transformer shown in Fig. 7. The prefault voltage at the fault location is 70 kV_{LL}, and the generator and transformer are not connected to the rest of the power system. The source impedances shown are the subtransient reactances (X_d") of the generator [9].

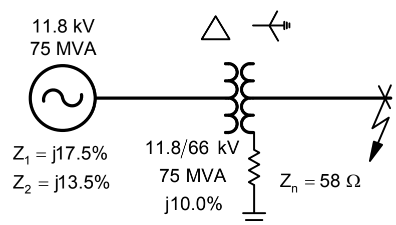


Fig. 7. One-line diagram for fault current calculations

III-a Select power and voltage bases for the per-unit system, and calculate current and impedance bases accordingly.

To use the per-unit system, first choose a power base and a voltage base. We choose a three-phase power base of 100 MVA and voltage bases as defined by the transformer:

$S_{base} = 100$ MVA
$V_{base\_delta} = 11.8$ kV
$V_{base\_wye} = 66$ kV

Notice that it is possible to have multiple voltage bases. We start by choosing one voltage base and then use the voltage ratios of the transformers to convert the original voltage base to all the other parts of the system. This means that at every transformer, there will be a voltage base conversion.

We then calculate the current and impedance bases using the power and voltage bases and (5) and (6). Depending on the voltage base that is active for the area we are working in, we will calculate different current and impedance bases.

On the delta side of the transformer, using V_{base_delta}:

$$I_{base\_delta} = \frac{S_{base}}{\sqrt{3} \cdot V_{base\_delta}} = \frac{100 \text{ MVA}}{\sqrt{3} \cdot 11.8 \text{ kV}} = 4.89 \text{ kA}$$

$$Z_{base\_delta} = \frac{\left(V_{base\_delta}\right)^2}{S_{base}} = \frac{(11.8 \text{ kV})^2}{100 \text{ MVA}} = 1.39\ \Omega$$

On the wye side of the transformer, using V_{base_wye}:

$$I_{base\_wye} = \frac{S_{base}}{\sqrt{3} \cdot V_{base\_wye}} = \frac{100 \text{ MVA}}{\sqrt{3} \cdot 66 \text{ kV}} = 874.77 \text{ A}$$

$$Z_{base\_wye} = \frac{\left(V_{base\_wye}\right)^2}{S_{base}} = \frac{(66 \text{ kV})^2}{100 \text{ MVA}} = 43.56\ \Omega$$

III-b Convert all impedances on the system as well as the prefault voltage to a common base.

To convert per-unit impedances from one base to another, use (8). The generator and transformer impedances are given in per unit on a 75 MVA, 11.8 kV base and need to be converted to a 100 MVA base.

$$Z_{G1} = j0.175 \frac{100 \text{ MVA}}{75 \text{ MVA}} \left(\frac{11.8 \text{ kV}}{11.8 \text{ kV}}\right)^2 = j0.233 \text{ pu}$$

$$Z_{G2} = j0.135 \frac{100 \text{ MVA}}{75 \text{ MVA}} \left(\frac{11.8 \text{ kV}}{11.8 \text{ kV}}\right)^2 = j0.180 \text{ pu}$$

$$Z_{T1} = Z_{T2} = Z_{T0} = j0.10 \frac{100 \text{ MVA}}{75 \text{ MVA}} \left(\frac{11.8 \text{ kV}}{11.8 \text{ kV}}\right)^2 = j0.133 \text{ pu}$$

The neutral impedance is given in ohms on the wye side of the transformer, so we need to divide it by the wye-side impedance base to convert it to per unit.

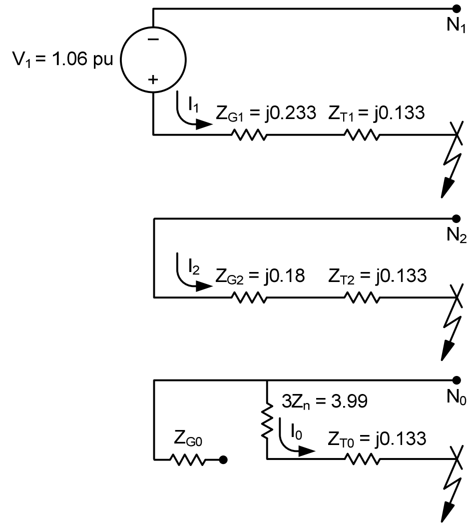
$$Z_n = \frac{58}{43.56} = 1.332 \text{ pu}$$

Convert the prefault voltage at the fault location to per unit by dividing by the wye-side voltage base.

$$V_f^{pre} = \frac{70 \text{ kV}}{66 \text{ kV}} = 1.06 \text{ pu}$$

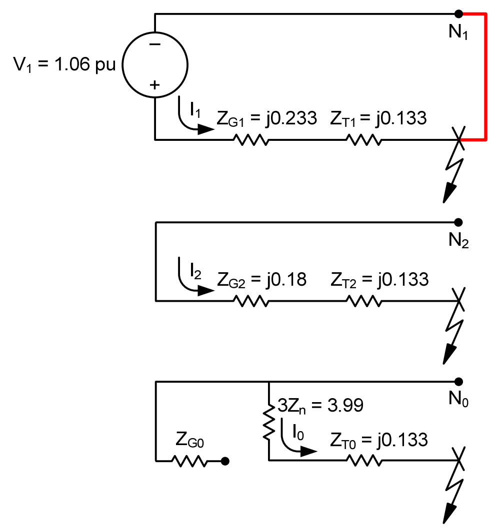
III-c Draw the positive-, negative-, and zero-sequence networks for this system up to the fault point.

The following figure shows the positive-, negative-, and zero-sequence networks for the system up to the fault location. The positive- and negative-sequence networks are similar, while the zero-sequence network has a break in it due to the delta connection of the transformer.



III-d What are the maximum short-circuit phase currents for a three-phase fault?

A three-phase fault is balanced and has no negative- or zero-sequence current. Therefore, only the positive-sequence network is connected, as shown in the following figure.



The positive-sequence current can be solved for by dividing the positive-sequence voltage by the impedances in the positive-sequence network.

$$I_1 = \frac{1.06}{j0.233 + j0.133} = -j2.89 \text{ pu or } 2.89\angle{-90} \text{ pu}$$

$$I_A = I_0 + I_1 + I_2 = 2.89\angle{-90} \text{ pu}$$

Convert the A-phase current from per unit to amperes by multiplying by the appropriate current base, I_{base_wye}.

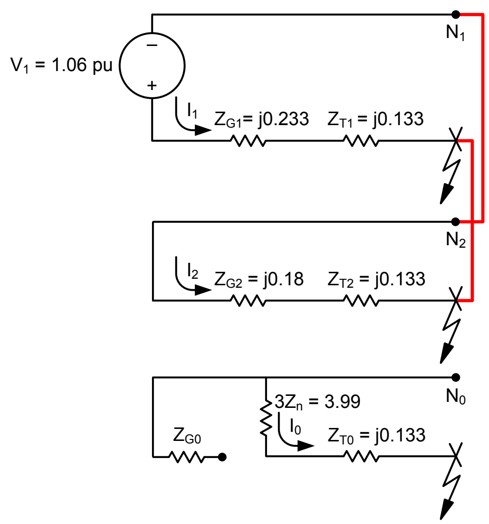
$$I_A = 2.89 \cdot 874.77 = 2528\angle{-90} \text{ A}$$

Because an ideal three-phase fault is balanced, $|I_A| = |I_B| = |I_C|$, and they are all 120 degrees out of phase, then:

$$I_B = 2528\angle 150 \text{ A}$$
$$I_C = 2528\angle 30 \text{ A}$$

III-e What are the maximum short-circuit phase currents for a B-phase-to-C-phase fault?

For a phase-to-phase fault, the positive- and negative-sequence networks are connected in parallel at the fault point, as shown in the following figure.



From this diagram, we can observe that $I_1 = -I_2$. We can solve for I_1 as follows:

$$I_1 = \frac{1.06}{j0.233 + j0.133 + j0.18 + j0.133} = 1.56\angle{-90} \text{ pu}$$
$$I_2 = 1.56\angle 90 \text{ pu}$$

$$I_A = I_0 + I_1 + I_2 = 0$$
$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2 = -2.7 \text{ pu}$$
$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2 = 2.7 \text{ pu}$$

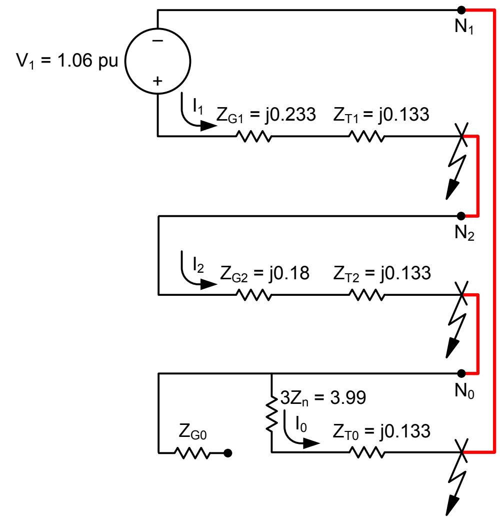
Convert the phase currents from per unit to amperes by multiplying by the appropriate current base, I_{base_wye}.

$$I_B = -2.7 \bullet 874.77 = 2361.8\angle 180 \text{ A}$$
$$I_C = 2.7 \bullet 874.77 = 2361.8\angle 0 \text{ A}$$

Notice that I_A is zero and I_B is 180 degrees out of phase with I_C, which is the ideal case for a phase-to-phase fault on the B- and C-phases.

III-f What are the maximum short-circuit phase currents for an A-phase-to-ground fault?

For a single-phase-to-ground fault, the three sequence networks are connected in series at the fault point, as shown in the following figure.



From this diagram, we can observe that $I_1 = I_2 = I_0$. We can solve for I_1 as follows:

$$I_1 = \frac{1.06}{j0.233 + j0.133 + j0.18 + j0.133 + j0.133 + 3.99}$$
$$= 0.2603\angle{-11.5} \text{ pu} = I_2 = I_0$$

$$I_A = I_0 + I_1 + I_2 = 3I_1 = 0.7809\angle{-11.5} \text{ pu}$$
$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2 = 0$$
$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2 = 0$$

Convert the A-phase current from per unit to amperes by multiplying by the appropriate current base, I_{base_wye}.

$$I_A = 0.7809 \bullet 874.77 = 683.1\angle{-11.5} \text{ A}$$

IV. Example 3: Fault Calculations for a Nonradial System

This example shows how to work in the per-unit system and calculate fault currents for a nonradial system, as shown in Fig. 8. The prefault voltage at the fault location is 1.05 per unit, and the load current is negligible. The source impedances shown are the subtransient reactances (X_d'') of the generators [3].

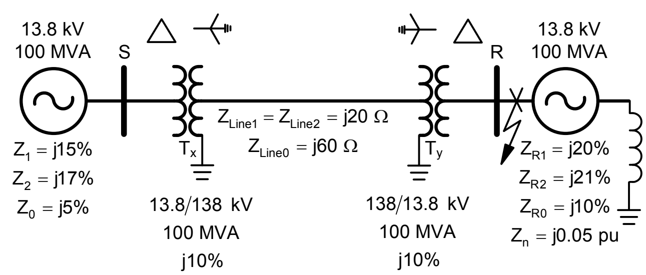


Fig. 8. One-line diagram of a nonradial system

IV-a Select power and voltage bases for the per-unit system, and calculate the current and impedance bases accordingly.

To use the per-unit system, first choose a power base and a voltage base. We choose a three-phase power base of 100 MVA and voltage bases as defined by the transformers:

S_{base} = 100 MVA
V_{base_line} = 138 kV
V_{base_buses} = 13.8 kV

Notice that it is possible to have multiple voltage bases. We start by choosing one voltage base and then use the voltage ratios of the transformers to convert the original voltage base to all the other parts of the system. This means that at every transformer, there will be a voltage base conversion.

We then calculate the current and impedance bases using the power and voltage bases along with (5) and (6). Depending on the voltage base that is active for the area we are working in, we will calculate different current and impedance bases.

On the line side of the transformers, using V_{base_line}:

$$I_{base\_line} = \frac{S_{base}}{\sqrt{3} \cdot V_{base\_line}} = \frac{100\text{ MVA}}{\sqrt{3} \cdot 138\text{ kV}} = 418.37\text{ A}$$

$$Z_{base\_line} = \frac{\left(V_{base\_line}\right)^2}{S_{base}} = \frac{(138\text{ kV})^2}{100\text{ MVA}} = 190.44\ \Omega$$

On the buses, which are on the delta sides of the transformers, using V_{base_buses}:

$$I_{base\_buses} = \frac{S_{base}}{\sqrt{3} \cdot V_{base\_buses}} = \frac{100\text{ MVA}}{\sqrt{3} \cdot 13.8\text{ kV}} = 4183.7\text{ A}$$

$$Z_{base\_buses} = \frac{\left(V_{base\_buses}\right)^2}{S_{base}} = \frac{(13.8\text{ kV})^2}{100\text{ MVA}} = 1.90\ \Omega$$

IV-b Convert all impedances on the system as well as the prefault voltage to a common base.

The generator and transformer impedances are already on the correct bases. The only impedances that need to be converted are the line impedances.

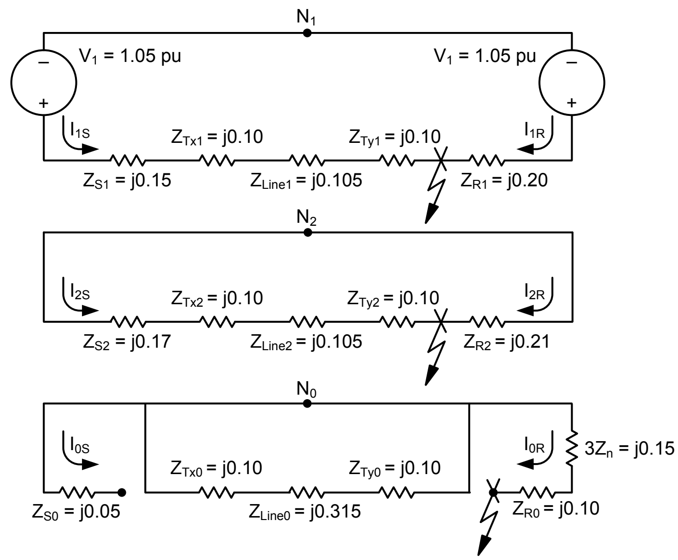
To convert the line impedances from ohms to per unit, divide them by Z_{base_line}.

$$Z_{Line1} = Z_{Line2} = \frac{j20}{190.44} = j0.105\text{ pu}$$

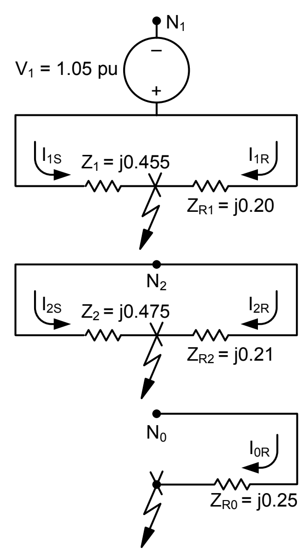
$$Z_{Line0} = \frac{j60}{190.44} = j0.315\text{ pu}$$

IV-c Draw the positive-, negative-, and zero-sequence networks for this system.

The following figure shows the positive-, negative-, and zero-sequence networks for the system. The positive- and negative-sequence networks are similar, while the zero-sequence network has two breaks in it due to the delta connections of the transformers.

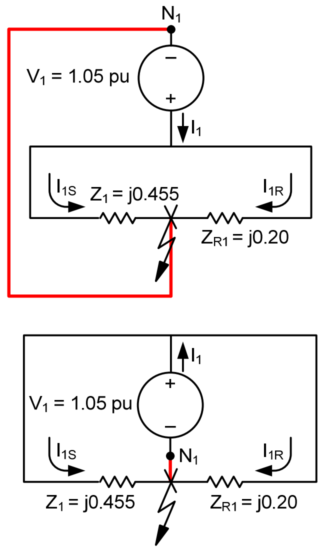


These networks can be simplified by combining the impedances on each side of the fault point, as shown in the following figure. The positive-sequence network is further simplified by combining both voltage sources into one equivalent source.



IV-d What are the maximum short-circuit phase currents for a three-phase fault?

A three-phase fault is balanced and has no negative- or zero-sequence current. Therefore, only the positive-sequence network is connected, as shown in the following figure.



The positive-sequence current through the fault can be solved as follows:

$$I_1 = I_{1S} + I_{1R}$$

$$I_1 = \frac{1.05}{j0.455} + \frac{1.05}{j0.20} = 7.557\angle{-90} \text{ pu}$$

For a three-phase fault, I_0 and I_2 are both 0.

Convert from the sequence to the phase domain as follows:

$$I_A = I_0 + I_1 + I_2 = 7.557\angle{-90} \text{ pu}$$

$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2 = 7.557\angle 150 \text{ pu}$$

$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2 = 7.557\angle 30 \text{ pu}$$

Convert the phase currents from per unit to amperes by multiplying by I_{base_buses}.

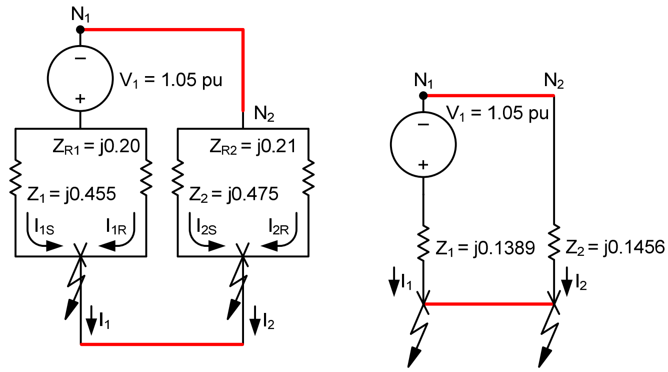
$$I_A = 7.557\angle{-90} \bullet 4183.7 = 31.62\angle{-90} \text{ kA}$$

$$I_B = 7.557\angle 150 \bullet 4183.7 = 31.62\angle 150 \text{ kA}$$

$$I_C = 7.557\angle 30 \bullet 4183.7 = 31.62\angle 30 \text{ kA}$$

IV-e What are the maximum short-circuit phase currents for a B-phase-to-C-phase fault?

For a phase-to-phase fault, the positive- and negative-sequence networks are connected in parallel at the fault point, as shown in the following figure. On the right side of the figure, the networks are represented by their equivalent impedances for simplification.



From this diagram, we can observe that $I_1 = -I_2$ and $I_0 = 0$. We can solve for I_1 as follows:

$$I_1 = \frac{1.05}{j0.1389 + j0.1456} = 3.69\angle{-90} \text{ pu}$$

$$I_2 = 3.69\angle 90 \text{ pu}$$

Convert from the sequence domain to the phase domain as follows:

$$I_A = I_0 + I_1 + I_2 = 0$$

$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2 = 6.39\angle 180 \text{ pu}$$

$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2 = 6.39\angle 0 \text{ pu}$$

Convert the phase currents from per unit to amperes by multiplying by I_{base_buses}.

$$I_A = 0$$

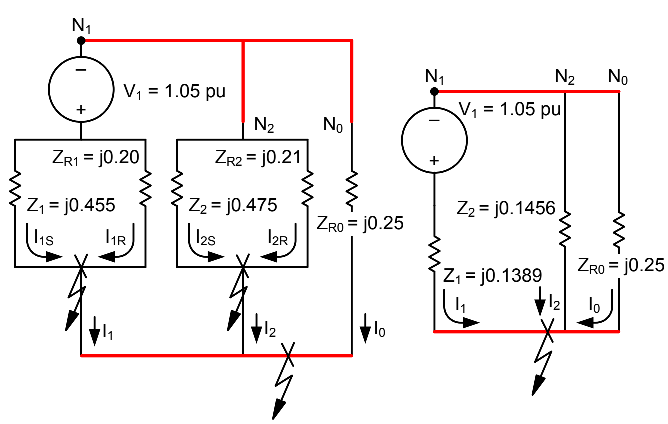
$$I_B = 6.39\angle 180 \bullet 4183.7 = 26.73\angle 180 \text{ kA}$$

$$I_C = 6.39\angle 0 \bullet 4183.7 = 26.73\angle 0 \text{ kA}$$

Notice that I_A is zero and I_B is 180 degrees out of phase with I_C, which is expected for a phase-to-phase fault on the B- and C-phases.

IV-f What are the maximum short-circuit phase currents for a B-phase-to-C-phase-to-ground fault?

For a double-line-to-ground fault, the three sequence networks are connected in parallel at the fault point, as shown in the following figure assuming zero fault resistance. On the right side of the figure, the networks are represented by their equivalent impedances for simplification.



From the diagram, we can solve for I_1 as follows:

$$I_1 = \frac{1.05}{j0.1389 + \left(j0.1456 \,||\, j0.25\right)} = 4.547\angle{-90} \text{ pu}$$

We can solve for the negative- and zero-sequence currents using a current divider and the positive-sequence current.

$$I_2 = -I_1\left(\frac{Z_{R0}}{Z_2 + Z_{R0}}\right) =$$

$$4.547\angle 90\left(\frac{j0.25}{j0.1456 + j0.25}\right) = 2.87\angle 90 \text{ pu}$$

$$I_0 = -I_1\left(\frac{Z_2}{Z_2 + Z_{R0}}\right) =$$

$$4.547\angle 90\left(\frac{j0.1456}{j0.1456 + j0.25}\right) = 1.67\angle 90 \text{ pu}$$

Convert from the sequence domain to the phase domain as follows:

$$I_A = I_0 + I_1 + I_2 = 0$$

$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2 = 6.90\angle 158.66 \text{ pu}$$

$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2 = 6.90\angle 21.33 \text{ pu}$$

Convert the phase currents from per unit to amperes by multiplying by I_{base_buses}.

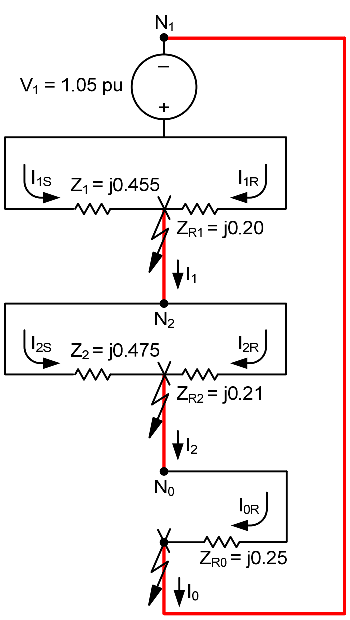
$$I_A = 0$$

$$I_B = 6.90\angle 158.66 \bullet 4183.7 = 28.85\angle 158.66 \text{ kA}$$

$$I_C = 6.90\angle 21.33 \bullet 4183.7 = 28.85\angle 21.33 \text{ kA}$$

IV-g What are the maximum short-circuit phase currents for an A-phase-to-ground fault?

For a single-phase-to-ground fault, the three sequence networks are connected in series at the fault point, as shown in the following figure.



From this diagram, we can see that $I_1 = I_2 = I_0$. We can solve for I_1 as follows:

$$I_1 = \frac{1.05}{\left(j0.455 \,||\, j0.20\right) + \left(j0.475 \,||\, j0.21\right) + j0.25} =$$

$$1.964\angle{-90} \text{ pu} = I_2 = I_0$$

Convert from the sequence domain to the phase domain as follows:

$$I_A = I_0 + I_1 + I_2 = 3I_2 = 5.893\angle{-90} \text{ pu}$$

$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2 = 0$$

$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2 = 0$$

Convert the phase currents from per unit to amperes by multiplying by I_{base_buses}.

$$I_A = 5.893 \bullet 4183.7 = 24.656\angle{-90} \text{ kA}$$

$$I_B = 0$$

$$I_C = 0$$

IV-h For an A-phase-to-ground fault, find the maximum positive-, negative-, and zero-sequence current contributions from Source S and Source R.

To find the contributions from Source S and Source R, perform a current divider using the sequence currents.

$$I_{1S} = I_1\left(\frac{j0.20}{j0.20 + j0.455}\right) = 0.5997\angle{-90} \text{ pu}$$

$$I_{1R} = I_1\left(\frac{j0.455}{j0.20 + j0.455}\right) = 1.364\angle{-90} \text{ pu}$$

$$I_{2S} = I_2\left(\frac{j0.21}{j0.21 + j0.475}\right) = 0.602\angle{-90} \text{ pu}$$

$$I_{2R} = I_2\left(\frac{j0.475}{j0.21 + j0.475}\right) = 1.362\angle{-90} \text{ pu}$$

$$I_{0S} = 0$$

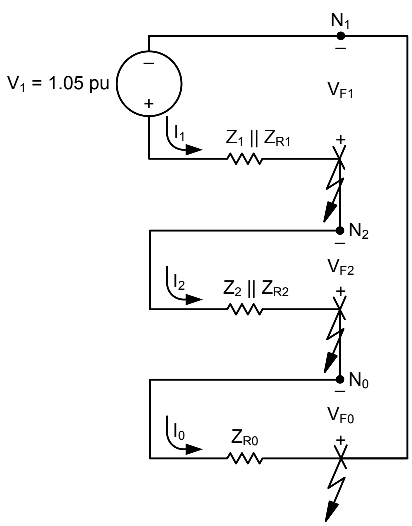
$$I_{0R} = I_0 = 1.964\angle{-90} \text{ pu}$$

IV-i Find the phase voltages at the fault location during an A-phase-to-ground fault.

First, find the sequence voltages at the fault location by writing voltage drop equations around each loop, as shown in the following equations and figure.

$$V_{F1} = V_1 - I_1 \left(Z_1 \parallel Z_{R1} \right)$$
$$V_{F2} = 0 - I_2 \left(Z_2 \parallel Z_{R2} \right)$$
$$V_{F0} = 0 - I_0 \left(Z_{R0} \right)$$

$$V_{F1} = 1.05 - \left(1.964\angle{-90}\right) \bullet \left(j0.1389\right) = 0.777 \text{ pu}$$
$$V_{F2} = 0 - \left(1.964\angle{-90}\right) \bullet \left(j0.1456\right) = -0.286 \text{ pu}$$
$$V_{F0} = 0 - \left(1.964\angle{-90}\right) \bullet \left(j0.25\right) = -0.491 \text{ pu}$$



Convert from the sequence domain to the phase domain as follows:

$$V_{FA} = V_{F0} + V_{F1} + V_{F2} = 0$$
$$V_{FB} = V_{F0} + \alpha^2 V_{F1} + \alpha V_{F2} = 1.178\angle{-128.66} \text{ pu}$$
$$V_{FC} = V_{F0} + \alpha V_{F1} + \alpha^2 V_{F2} = 1.178\angle 128.66 \text{ pu}$$

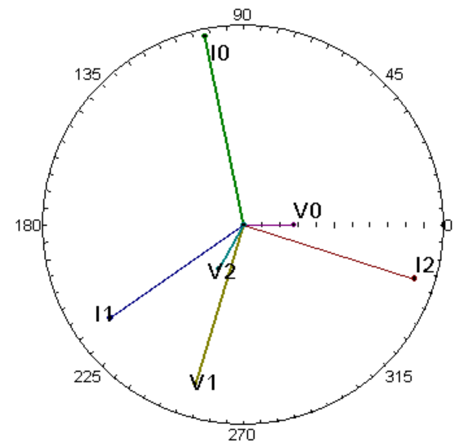
V. Example 4: Changing Bases

This example shows the importance of using the right base when computing symmetrical components. Typical textbook examples use an A-phase base, which always assumes an A-phase-to-ground, B-phase-to-C-phase, B-phase-to-C-phase-to-ground, or three-phase fault. For other fault types, the base will need to be changed accordingly in order to compute the correct symmetrical components.

This example shows a B-phase-to-ground fault that occurred on a transmission line. Open the event record titled **Example 4.cev**, and view the symmetrical components during the fault.

V-a Are the symmetrical component currents what we expect to see for a phase-to-ground fault?

Because the sequence networks are connected in series for a phase-to-ground fault, we expect to see $I_1 = I_2 = I_0$. For this event, the symmetrical components are each 120 degrees out of phase with each other, as shown in the following figure. This is not correct.



V-b Derive the symmetrical components for an A-phase-to-ground fault.

Typical textbooks as well as the introduction to this paper use (1) to derive symmetrical components from phase quantities. This method assumes an ABC system phase rotation as well as an A-phase reference or base. An A-phase base means that the A-phase is in the top position of the phase current matrix followed by B-phase and C-phase for a system with an ABC phase rotation. An A-phase base is only valid for A-phase-to-ground, B-phase-to-C-phase, B-phase-to-C-phase-to-ground, or three-phase faults.

Because an A-phase-to-ground fault assumes $I_B = I_C = 0$, then:

$$I_0 = I_1 = I_2 = \frac{1}{3} I_A$$

This results in the zero-, positive-, and negative-sequence currents being equal and in phase with each other, which is what we expect. Equation (1) works fine when the fault is an A-phase-to-ground fault.

V-c Derive the symmetrical components for a B-phase-to-ground fault.

Use (1) to calculate symmetrical components for a B-phase-to-ground fault.

Because a B-phase-to-ground fault assumes $I_A = I_C = 0$, then:

$$I_0 = \frac{1}{3} I_B$$
$$I_1 = \frac{1}{3} \alpha I_B$$
$$I_2 = \frac{1}{3} \alpha^2 I_B$$

The unexpected result is that I_0, I_1, and I_2 are 120 degrees out of phase instead of in phase with each other. This matches the phasors in the figure from the answer to Question V-a and is incorrect, which proves that (1) does not work for a B-phase-to-ground fault.

V-d How do we obtain the correct symmetrical component values for a B-phase-to-ground fault?

To correctly calculate the symmetrical components for something other than the typical A-phase base faults, we must change the base in (1). This is done by rotating the terms in the phase current matrix so that the top position is the reference, the middle term lags the reference by 120 degrees, and the bottom term leads the reference by 120 degrees.

For a B-phase base on a system with an ABC system phase rotation, the new equation is as follows:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_B \\ I_C \\ I_A \end{bmatrix}$$

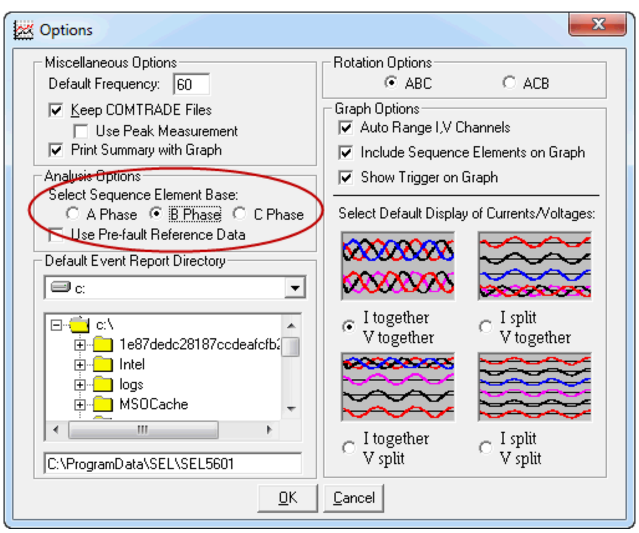
Using this new transformation equation and assuming that $I_A = I_C = 0$ for a B-phase-to-ground fault, we obtain:

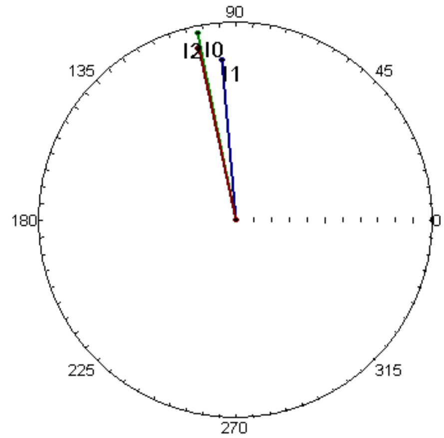
$$I_0 = I_1 = I_2 = \frac{1}{3} I_B$$

Notice that I_0, I_1, and I_2 are all in phase with each other, which is what we expect to see for a phase-to-ground fault.

V-e Why did the symmetrical components in ACSELERATOR Analytic Assistant not calculate correctly?

The event viewer needs to know what base to use when calculating symmetrical components. If the correct base is not selected, the symmetrical components will calculate incorrectly, as we demonstrated in this example. The following figures show that selecting **B Phase** as the base in ACSELERATOR Analytic Assistant will make the sequence phasors come in line with each other.





When viewing symmetrical components in ACSELERATOR Analytic Assistant, it is very important to always select the correct base. For a single-phase-to-ground fault, the correct base is the faulted phase. For a phase-to-phase or double-line-to-ground fault, the correct base is the unfaulted phase. For a three-phase fault, the base selection does not matter.

VI. Example 5: Phase Rotation

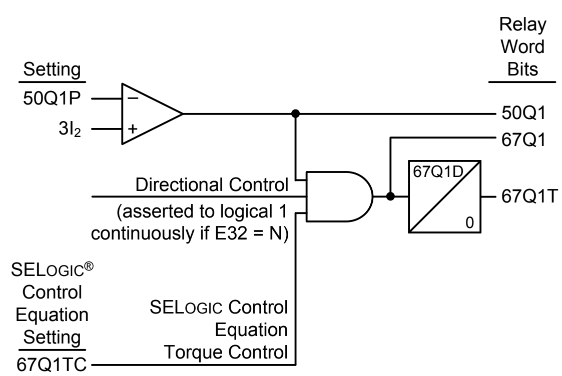
This example shows the importance of phase rotation when calculating sequence quantities. The event titled **Example 5.cev** is a simulated load condition on an SEL-351S Protection System. The trip equation in the relay is:

TR =51P1T + 51G1T + 67P1 + 50Q1 + OC

where 50Q1 is a negative-sequence instantaneous overcurrent element.

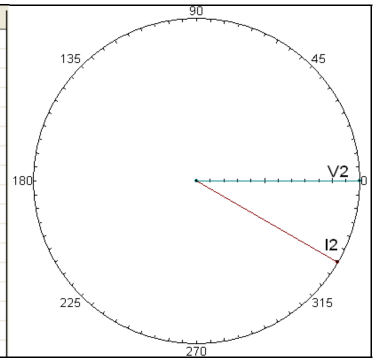
VI-a What is the pickup setting for 50Q1 in the relay? Based on the negative-sequence current seen in the event, should the relay have tripped?

From the relay settings, we can see that 50Q1P = 3 A secondary. The following figure is from the SEL-351S Instruction Manual and shows that the 50Q1 element asserts when 3 • I_2 becomes greater than the 50Q1P setting.



The phasors in the event show that I_2 is about 600 A primary, as shown in the following figure.

Channel	Mag	Angle	Scale	Show	Ref
IA	599.1	330.0	1	1	0
IB	599.2	90.0	1	1	0
IC	599.9	210.1	1	1	0
IN	0.3	272.1	1	0	0
IG	1.9	231.6	1	0	0
VA(kV)	11.8	0.0	1	1	1
VB(kV)	11.9	120.0	1	1	0
VC(kV)	11.9	240.0	1	1	0
VS(kV)	0.0	250.0	1	0	0
VDC	22.0	N/A	1	0	0
FREQ	60.0	N/A	1	0	0
I0	0.3	250.0	1	1	0
I1	0.3	137.5	1	1	0
I2	599.4	330.1	1	1	0
V0	0.0	208.5	1	1	0
V1	0.0	168.2	1	1	0
V2	11.8	0.0	1	1	0



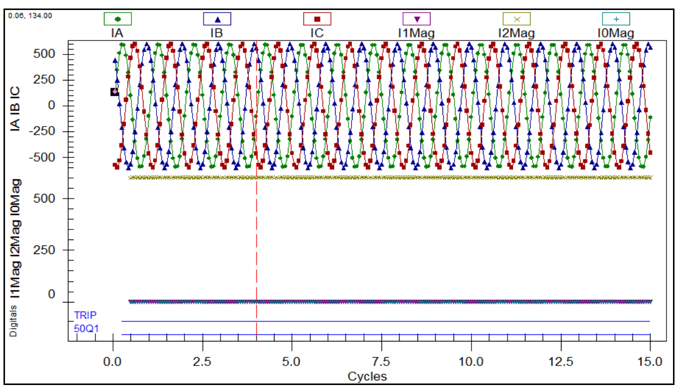
With a current transformer (CT) ratio of 120, the measured 3 • I_2 comes out to 15 A secondary.

$$3I_2 = 3 \bullet 599.4 = 1798 \text{ A primary}$$

$$\frac{1798 \text{ A primary}}{120} \cong 15 \text{ A secondary}$$

This is greater than the pickup value of 3 A secondary, so it looks as if the relay should have tripped for this condition.

The following event report shows that although the negative-sequence current magnitude is significantly above the pickup, the 50Q1 element did not assert and the relay did not issue a trip.



VI-b Using the phase currents from the event, calculate the negative-sequence current I_2.

To convert phase currents to symmetrical components, use (1). Using the phase currents from the previous figure, calculate the negative-sequence component.

$$I_2 = \frac{1}{3}\left(I_A + \alpha^2 I_B + \alpha I_C\right)$$

$$I_2 = \frac{1}{3}\left[(599.1\angle 330) + \alpha^2 (599.2\angle 90) + \alpha (599.9\angle 210.1)\right]$$

$$= 599.4\angle{-30} \text{ A}$$

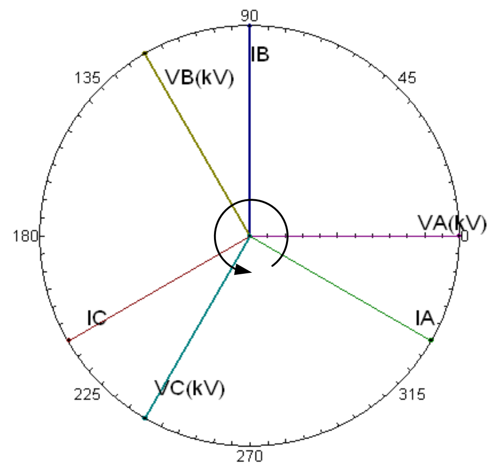
This matches the negative-sequence current shown in the second figure in the answer to Question VI-a.

VI-c Is it normal to see this much negative-sequence current during unfaulted conditions?

No. A large amount of negative-sequence current or voltage that appears in normal load metering along with a very small amount of positive-sequence current or voltage is cause for suspicion.

VI-d What is the phase rotation of the system? Does this match the phase rotation setting in the relay?

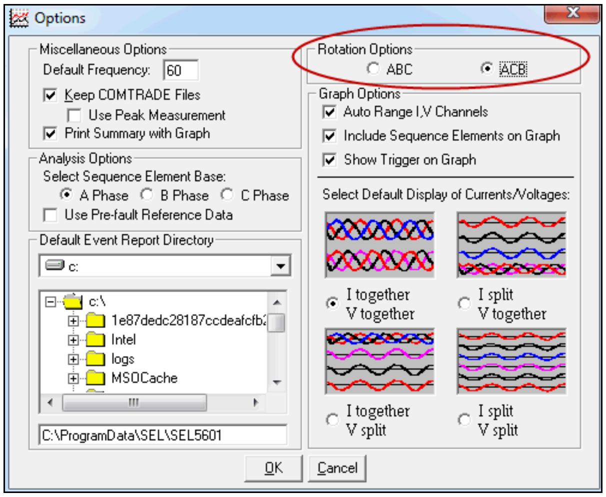
A large amount of negative-sequence current with a small amount of positive-sequence current seen during normal conditions is normally a system phase rotation issue. The following figure shows the phasors during the event. As the phasors rotate in a counterclockwise direction, the order in which they pass a reference point is A-phase, then C-phase, and then B-phase. This means the system has an ACB phase rotation.



The settings in the relay also show that the global setting PHROT is set to ACB, which is correct and matches the event phasors.

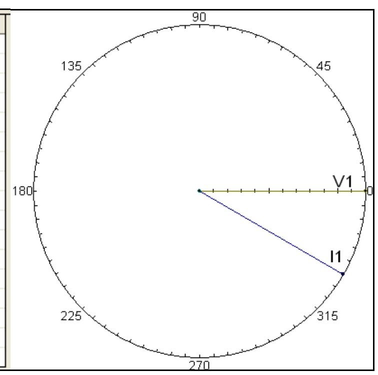
VI-e Why is ACSELERATOR Analytic Assistant calculating high negative-sequence quantities?

When viewing events in ACSELERATOR Analytic Assistant, the symmetrical components of the voltages and currents that are displayed are calculated based on the phase quantities. Because of this, we must tell the software the phase rotation of the system, which can be done under the **Options** menu, as shown in the following figure.



The following figure shows the symmetrical components after changing the phase rotation in ACSELERATOR Analytic Assistant to ACB. Notice the negative-sequence current is now very small and the positive-sequence current is very high. This is the opposite of the results we saw when we assumed an ABC system phase rotation.

Channel	Mag	Angle	Scale	Show	Ref
IA	599.1	330.0	1	1	0
IB	599.2	90.0	1	1	0
IC	599.9	210.1	1	1	0
IN	0.3	272.1	1	0	0
IG	1.9	231.6	1	0	0
VA(kV)	11.8	0.0	1	1	1
VB(kV)	11.9	120.0	1	1	0
VC(kV)	11.9	240.0	1	1	0
VS(kV)	0.0	250.0	1	0	0
VDC	22.0	N/A	1	0	0
FREQ	60.0	N/A	1	0	0
I0	0.3	250.0	1	1	0
I1	599.4	330.0	1	1	0
I2	0.6	167.5	1	1	0
V0	0.0	208.5	1	1	0
V1	11.8	0.0	1	1	0
V2	0.0	128.4	1	1	0



VI-f Calculate the negative-sequence current by hand using ACB phase rotation.

For ACB phase rotation, (1) needs to be modified by putting the currents in the phase matrix in ACB order, as follows:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_A \\ I_C \\ I_B \end{bmatrix}$$

$$I_2 = \frac{1}{3}\left(I_A + \alpha^2 I_C + \alpha I_B\right)$$

$$I_2 = \frac{1}{3}\left[\left(599.1\angle 330\right) + \alpha^2\left(599.9\angle 210.1\right) + \alpha\left(599.2\angle 90\right)\right]$$

$$= 1.36\angle 147 \text{ A}$$

Notice that this does not exactly match the results for I_2 in the second figure in the answer to Question VI-e. This is due to a rounding error that comes into play because I_2 has such a small magnitude (0.6 A primary). The accuracy of the phase currents (I_A, I_B, and I_C) we are using in our hand calculations is limited to the number of significant digits displayed by the software. The difference between the hand-calculated results and the ACSELERATOR Analytic Assistant results is due to the fact that ACSELERATOR Analytic Assistant is actually using more significant digits for the phase currents. The important thing to note is that when the correct phase rotation is used, the traditional method matches ACSELERATOR Analytic Assistant and results in extremely small (and negligible) values of negative-sequence current.

VI-g Why did the relay not trip?

The relay did not trip because there was very little negative-sequence current present. The relay was calculating the negative sequence correctly because it knew the phase rotation was ACB (setting PHROT = ACB). ACSELERATOR Analytic Assistant, however, needs to be told the correct phase rotation in order to calculate the symmetrical components correctly.

VII. Example 6: Fault Locator

This example shows how to use symmetrical components to determine a fault location using event reports from two ends of a transmission line. An internal single-line-to-ground fault was detected on a transmission line by the relays at both ends, as shown in Fig. 9. The event reports from each relay are provided in the event records titled **Example 6 - Side S.eve** and **Example 6 - Side R.eve**.

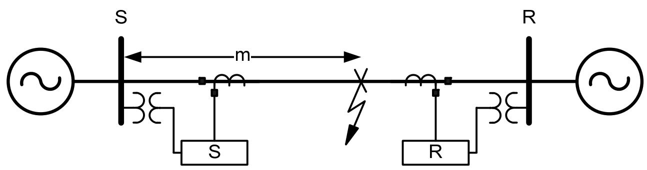
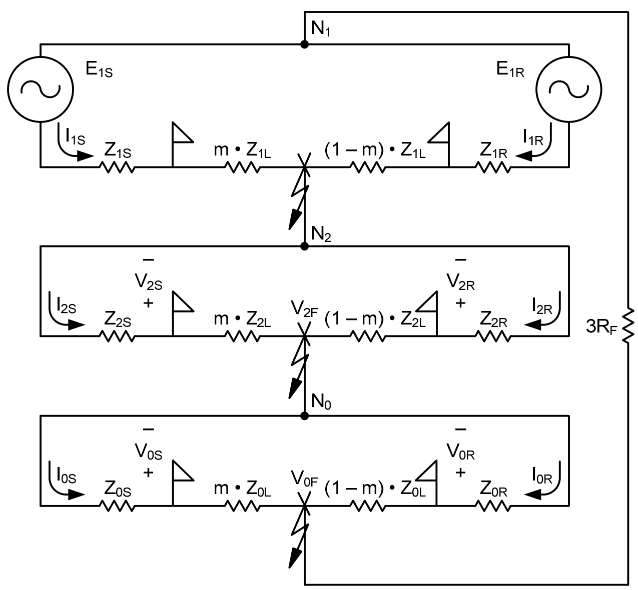


Fig. 9. Fault location on a two-source power system

VII-a Draw the sequence networks for this fault.

Because the fault is a line-to-ground fault, the sequence networks are connected in series at the fault point, as shown in the following figure. The distance to the fault in per unit of total line length is *m*. The flags mark the relay positions.



VII-b Using the sequence networks, write an equation to solve for the fault location *m*.

Any of the sequence networks can be used to solve for the fault location, but the negative-sequence network is preferred because it is not affected by load flow or zero-sequence mutual coupling.

To solve for *m*, write two voltage drop equations at the V_{2F} fault location—one between node V_{2F} and the S relay and one between node V_{2F} and the R relay. These equations are as follows:

$$V_{2S} - I_{2S} \bullet m \bullet Z_{2L} = V_{2F}$$

$$V_{2R} - I_{2R} \bullet (1 - m) \bullet Z_{2L} = V_{2F}$$

Because we have event reports from both ends of the line, the negative-sequence voltages and currents as well as the negative-sequence line impedance (from the relay settings) are known. This results in two equations and two unknowns (V_{2F} and *m*).

Because both equations are equal to V_{2F}, we can eliminate this unknown variable by setting the equations equal to each other.

$$V_{2S} - I_{2S} \bullet m \bullet Z_{2L} = V_{2R} - I_{2R} \bullet (1 - m) \bullet Z_{2L}$$

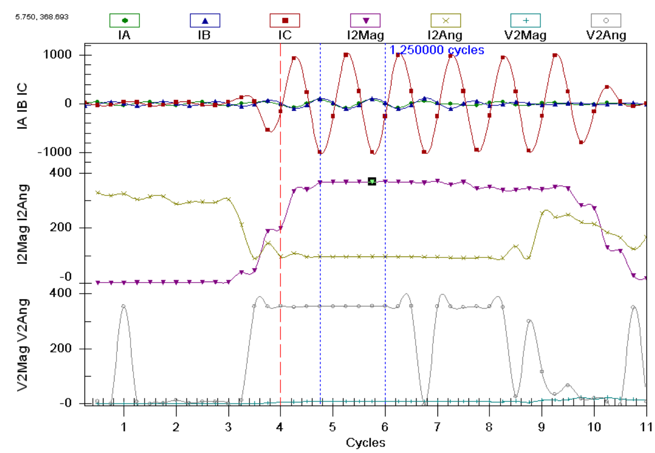
Then rearrange to solve for *m*.

$$V_{2S} - V_{2R} + I_{2R} \bullet Z_{2L} = m\left(I_{2S} \bullet Z_{2L} + I_{2R} \bullet Z_{2L}\right)$$

$$m = \frac{V_{2S} - V_{2R} + I_{2R} \bullet Z_{2L}}{Z_{2L}\left(I_{2S} + I_{2R}\right)}$$

VII-c Use the event reports to obtain voltage and current values during the fault as well as the negative-sequence line impedance. Solve for *m*.

V_{2S} and I_{2S} (magnitude and angle) can be found from the **Example 6 – Side S.eve** event during the time of the fault. It is best to select values that are stable and unchanging. In this event, stable data are found between 4.75 and 6.75 cycles, as shown between the two dashed blue vertical lines in the following figure.



Channel	Mag	Angle	Scale	Show	Ref
IPOL	0.0	156.4	1	0	0
IR	841.5	94.7	1	0	0
IA	86.4	296.7	1	1	0
IB	107.9	280.7	1	1	0
IC	1030.8	97.1	1	1	0
VA	92.1	54.9	1	1	0
VB	92.1	289.5	1	1	0
VC	74.8	171.1	1	1	0
I0	280.8	94.7	1	0	0
I1	381.7	99.1	1	1	0
I2	368.7	96.9	1	1	0
V0	3.3	0.0	1	1	1
V1	86.3	171.9	1	1	0
V2	8.2	355.6	1	1	0

Because this is a C-phase-to-ground fault, we must select values on a C-phase base.

From this event, we gather the following:

$$I_{2S} = 368.7\angle 96.9 \text{ A}$$

$$V_{2S} = 8.2\angle 355.6 \text{ kV}$$

V_{2R} and I_{2R} (magnitude and angle) can be found in a similar way from the **Example 6 – Side R.eve** event during the time of the fault.

Channel	Mag	Angle	Scale	Show	Ref
IPOL	0.0	262.4	1	0	0
IR	2667.2	95.1	1	0	0
IA	121.7	133.5	1	1	0
IB	119.6	84.8	1	1	0
IC	2438.2	93.7	1	1	0
VA	89.9	56.7	1	1	0
VB	89.1	291.6	1	1	0
VC	48.5	170.9	1	1	0
I0	883.5	95.0	1	1	0
I1	749.8	92.4	1	1	0
I2	805.3	93.5	1	1	0
V0	11.4	0.0	1	1	1
V1	75.8	173.5	1	1	0
V2	16.0	356.5	1	1	0

From this event, we gather the following:

$$I_{2R} = 805.3\angle 93.5 \text{ A}$$

$$V_{2R} = 16.0\angle 356.5 \text{ kV}$$

From the event report relay settings, we can find that R1 = R2 = 16.77 and X1 = X2 = 65.21 (note that if these settings are in ohms secondary, they must be converted to ohms primary). Converting from the rectangular form of 16.77 + j65.21 to the polar form, we obtain the following:

$$Z_{2L} = 67.33\angle 75.58\ \Omega$$

We can then plug these data into the following equation to solve for the fault location *m*.

$$m = \frac{V_{2S} - V_{2R} + I_{2R} \bullet Z_{2L}}{Z_{2L}\left(I_{2S} + I_{2R}\right)}$$

$$m = \frac{(8200\angle 355.6) - (16000\angle 356.5) + (805.3\angle 93.5) \bullet (67.33\angle 75.58)}{67.33\angle 75.58(368.7\angle 96.9 + 805.3\angle 93.5)}$$

$$= 0.78 \text{ pu}$$

From the LL setting in the relay, we see that the line length is 82 miles. 0.78 • 82 miles gives us a fault location of 63.96 miles from Side S.

Now that the fault location *m* is known, it is possible to use the same sequence networks to solve for the fault resistance, if desired. The figure in the answer to Question VII-a will now have all known impedances, with the exception of the fault resistance, $3R_F$. For more information on fault location algorithms and symmetrical components, see [10], [11], and [12].

VIII. Example 7: Transformer Line-to-Ground Fault

This example shows how to derive the phase shift, symmetrical components, and fault currents across a delta-wye transformer. The event report titled **Example 7.cev** was generated after a current differential relay protecting a Dy1 transformer tripped, as shown in Fig. 10. Although the misoperation of the relay is not the focus of this exercise, it was caused by incorrect winding current compensation settings in the relay.

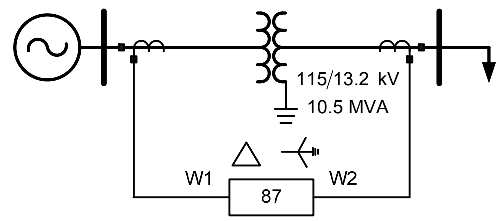
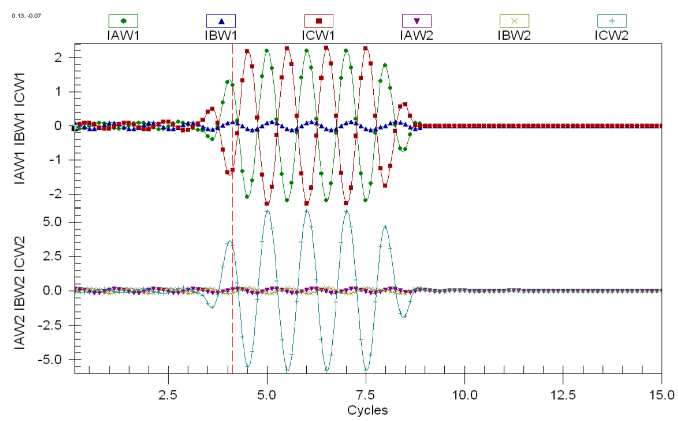


Fig. 10. Transformer current differential relay protecting a Dy1 transformer

VIII-a What type of fault is this? Assuming a radial system, is the fault internal or external to the zone of protection?

It is a C-phase-to-ground fault on the wye side of the transformer. The fault is external because both relay CTs see fault current. The following figure shows the waveform for an external C-phase-to-ground fault.



VIII-b Do we expect the prefault currents on the delta side to lead or lag the currents on the wye side?

The example states that it is a Dy1 transformer. This standard means that the delta side leads the wye side by (1 • 30) = 30 degrees for the prefault balanced phasors.

VIII-c The transformer is connected to the system as shown in Fig. 11. Does this change the current lead/lag relationship we expect to see across the transformer? If so, how?

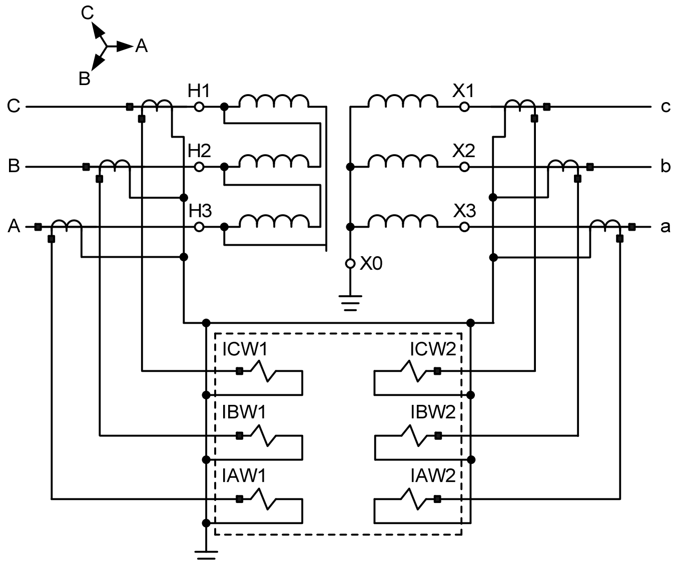


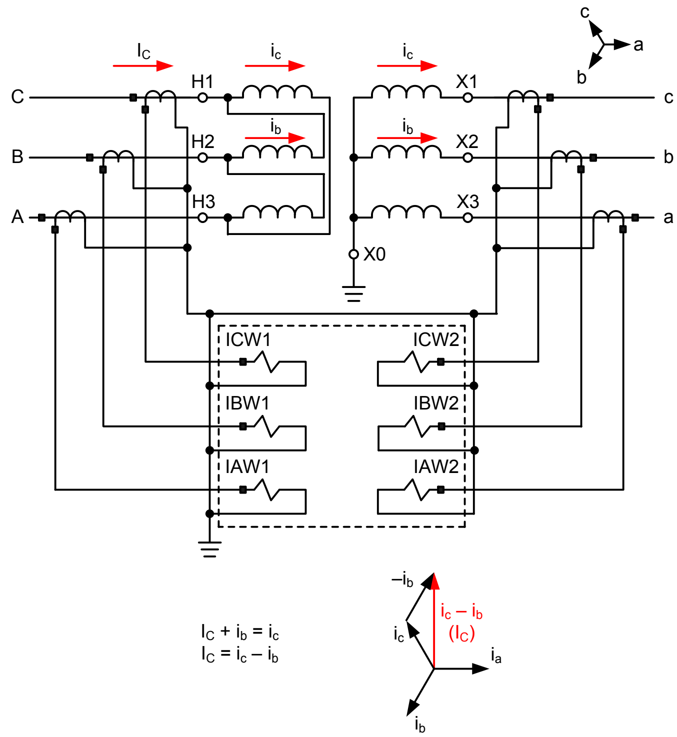
Fig. 11. Transformer phase-to-bushing connections

The standard assumption that a Dy1 transformer delta side leads the wye side by 30 degrees is only valid when three conditions are true:

1. The system has an ABC phase rotation.
2. The A-phase on the system goes to H1 and X1 on the transformer, the B-phase on the system goes to H2 and X2 on the transformer, and the C-phase on the system goes to H3 and X3 on the transformer.
3. The A-phase on the system goes to the A-phase on the relay, the B-phase on the system goes to the B-phase on the relay, and the C-phase on the system goes to the C-phase on the relay.

In this case, Condition 2 is not true. This means that the standard of delta leading wye by 30 degrees is not necessarily true.

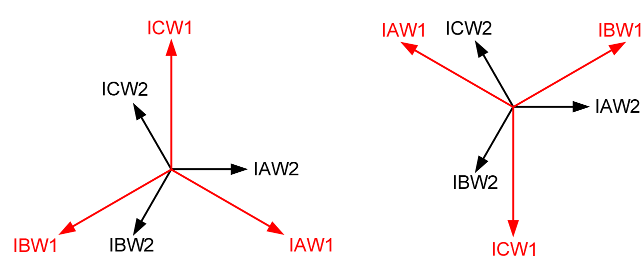
We can easily trace through any transformer connection to derive the lead/lag relationship between currents on either side. First, assume current flow through the transformer from the delta side to the wye side. Knowing that the individual phase windings of the delta side are magnetically coupled to the individual windings of the wye side and assuming a transformer ratio of 1:1, we can conclude that the currents through them are the same. We can then write KCL equations to derive the currents on the phases coming into the delta winding. The derivation of this KCL equation for the delta-side C-phase current is shown in the following figure.



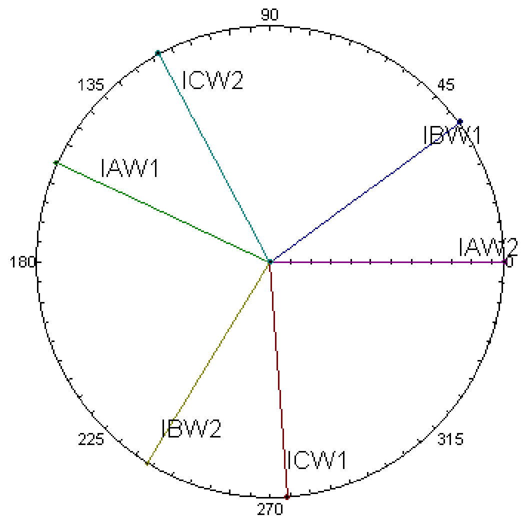
The result shows that the high-side (delta) currents are $\sqrt{3}$ larger and lag the low-side (wye) currents by 30 degrees. The positive-sequence currents reflect the same behavior.

VIII-d Draw the phasors for the prefault currents we expect to see on the system as well as the currents coming into the relay. Do these match the prefault phasors in the event?

The prefault current phasors are shown in the following figure, with Winding 1 being the delta side and Winding 2 being the wye side. The left diagram shows the prefault currents seen on the system, and the right diagram shows the prefault currents seen by the relay. Note that the Winding 1 currents are 180 degrees out of phase from the Winding 2 currents when seen by the relay because the CT polarity of Winding 1 is opposite that of Winding 2.

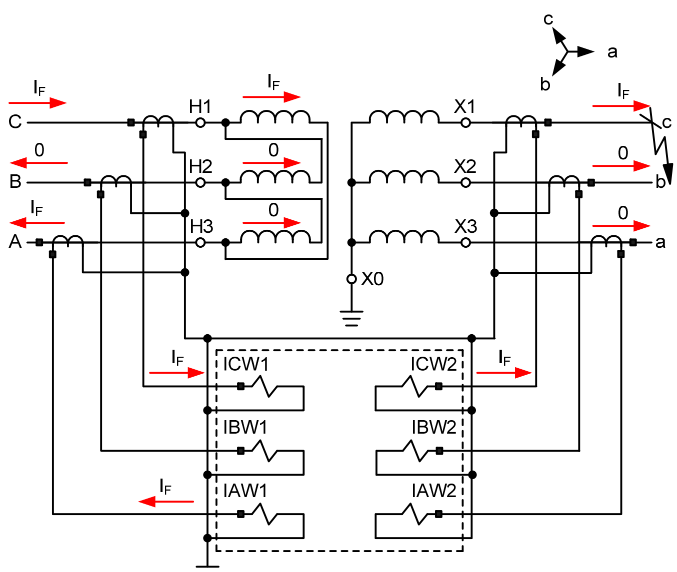


The phasors seen by the relay match the event, as shown in the following figure.

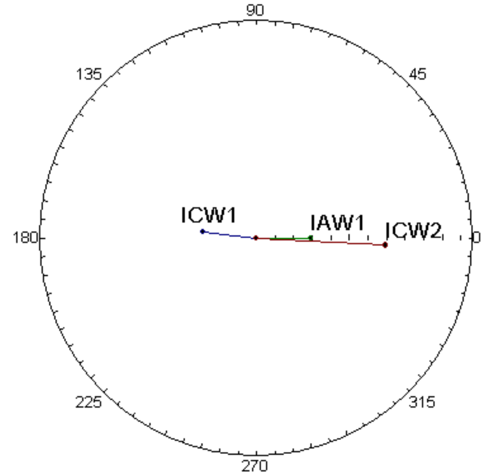


VIII-e Draw the phasors that we expect to see on the system as well as the phasors coming into the relay during the fault. Does this match what the event shows?

Working from right to left, we can trace the fault through the transformer in a way similar to what we did with the prefault currents. This is shown in the following figure, assuming load is negligible on the unfaulted phases and the transformer ratio is 1:1.

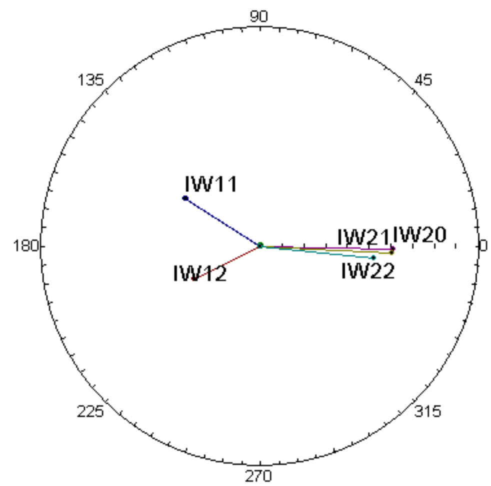


The phasors in the event, shown in the following figure, match what we expect to see. IAW1 and ICW1 are equal in magnitude and 180 degrees out of phase. ICW1 and ICW2 are also 180 degrees out of phase.

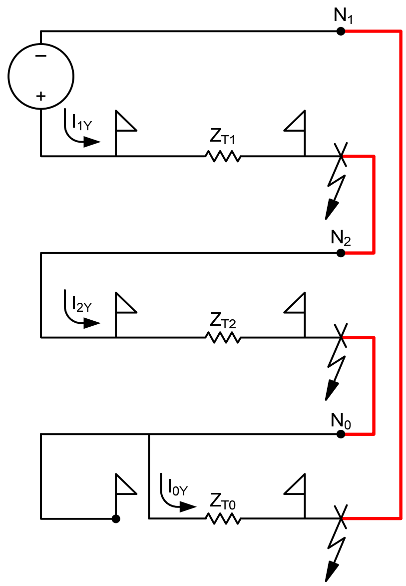


VIII-f Look at the symmetrical components in the event. Derive these phasors by drawing the sequence network of the fault.

The sequence current phasors during the fault are shown in the following figure. Note that we must select a C-phase base when viewing the sequence phasors because this is a C-phase-to-ground fault.



The following figure shows the sequence network diagram for this fault, with the flags marking the CT locations. Because this is a phase-to-ground fault, all three sequence networks are connected in series. From this diagram, we notice that there is no zero-sequence current flowing through the CT on the delta side. This is why there is no zero-sequence current phasor for Winding 1 in the previous figure.



The phase currents used by the 87 elements in the relay are a combination of positive-, negative-, and zero-sequence currents. If the relay has wye CTs, we must remove the zero-sequence current from Winding 2 inside the relay because the zero-sequence current is removed from Winding 1 due to the delta connection of the transformer. If we use a delta CT on the Winding 2 side, this would have the same effect. A restricted earth fault (REF) element or ground relay on the X0 bushing can be used in order to detect sensitive ground faults on the wye side of the transformer.

It is important to note that this network represents a phase-to-ground fault on the wye side of the transformer. This means we need to solve for the sequence currents going through the wye side first and then apply a delta transformation to solve for the currents going through the delta side.

Because this is a C-phase-to-ground fault, we need to use a C-phase base, so our transformation equation becomes (with an ABC system phase rotation):

$$\begin{bmatrix} I_{0Y} \\ I_{1Y} \\ I_{2Y} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_{CY} \\ I_{AY} \\ I_{BY} \end{bmatrix}$$

$$I_{0Y} = \frac{1}{3}\left(I_{CY} + I_{AY} + I_{BY}\right)$$

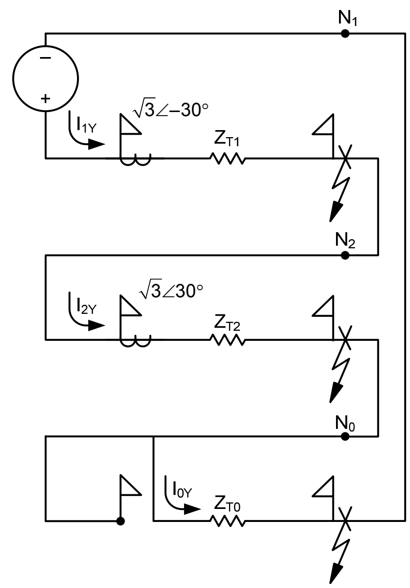
$$I_{AY} = I_{BY} = 0$$

$$I_{0Y} = \frac{1}{3} I_{CY}$$

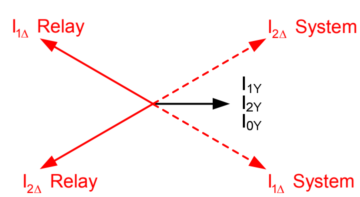
$$I_{0Y} = I_{1Y} = I_{2Y}$$

This solves for the sequence currents on the wye side of the transformer. In the first figure in this answer, we can see that all three sequence phasors on the wye side (Winding 2) are equal in magnitude and phase, which is expected from the equations we derived. We can also see that the magnitude of the sequence phasors is about 1/3 that of the C-phase current on Winding 2.

Earlier, we derived that the currents on the delta side of the transformer are going to be $\sqrt{3}$ greater in magnitude and lagging the wye-side currents by 30 degrees in the positive sequence (leading by 30 degrees in the negative sequence). This is represented in the following figure by taking the currents through the networks and transforming them through a CT that applies the appropriate magnitude increase and angle shift. The sequence currents seen on the delta side are after the CT transformation.



The sequence phasors that we expect to see on the system are shown in the following figure. Notice that $I_{1\Delta}$ is $\sqrt{3}$ greater in magnitude and lags I_{1Y} by 30 degrees, while $I_{2\Delta}$ is $\sqrt{3}$ greater in magnitude and leads I_{2Y} by 30 degrees. Due to the CT polarity, we rotate the delta currents on the system by 180 degrees to see what the relay sees.



This matches the sequence phasors from the event, as shown in the first figure in this answer. Note that in the event, the delta currents are not $\sqrt{3}$ greater in magnitude than the wye currents, as we expect. This is because the $\sqrt{3}$ difference is based on an assumed transformer turns ratio of 1:1. The transformer turns ratio is calculated as:

$$n = \frac{\frac{13.2\text{ kV}}{\sqrt{3}}}{115\text{ kV}} = 0.0662$$

We can now expect the delta currents to be $n\sqrt{3}$ greater than the wye currents, or 0.1147 times greater. It is also important to note that this relationship is true for amperes primary, so we need to convert the amperes secondary given in the event to amperes primary.

For example, we expect $I_{1\Delta}$ to be 0.1147 multiplied by I_{1Y} in amperes primary. The event report shows $I_{1\Delta}$ = 1.3 A secondary and I_{1Y} = 2.0 A secondary during the fault. Multiplying each winding current by its CT ratio (40:1 for the delta winding and 240:1 for the wye winding), we obtain $I_{1\Delta}$ = 52 A primary and I_{1Y} = 480 A primary. I_{1Y} (480 A) multiplied by 0.1147 gives approximately $I_{1\Delta}$ (52 A).

VIII-g Using the sequence components, work backwards to derive the phase fault currents on the delta and wye sides of the transformer.

Because we are using the C-phase as a base, our transformation equation is as follows:

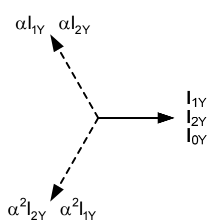
$$\begin{bmatrix} I_C \\ I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$I_C = I_0 + I_1 + I_2$$

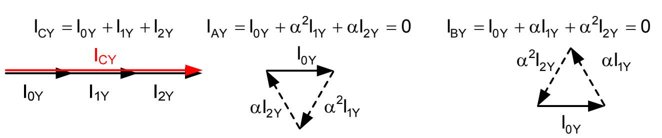
$$I_A = I_0 + \alpha^2 I_1 + \alpha I_2$$

$$I_B = I_0 + \alpha I_1 + \alpha^2 I_2$$

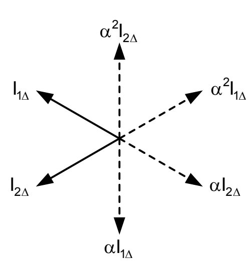
These computations can be done mathematically or graphically. The graphical method is shown here. For the wye currents, we first draw the sequence phasors shifted by α and α^2 that we will need, as shown in the following figure.



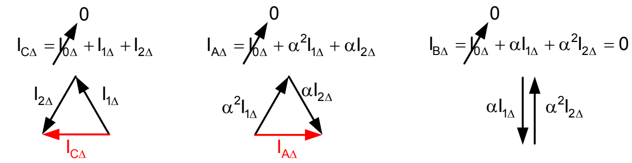
We then apply the transformations, as shown in the following figure.



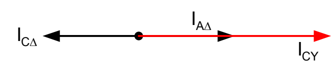
For the delta currents, we first draw the sequence phasors shifted by α and α^2 that we will need, as shown in the following figure.



We then apply the transformations, as shown in the following figure.



Combining the results of the phase currents on the delta side and wye side, we get the phasors shown in the following figure. This matches the phase currents during the event, as shown in the last figure in the answer to Question VIII-e.



IX. Example 8: Transformer Phase-to-Phase Fault

This example shows how to derive the phase shift, symmetrical components, and fault currents across a delta-wye transformer. The event report titled **Example 8.txt** was generated after a current differential relay protecting a delta-wye transformer tripped, as shown in Fig. 12. The misoperation of the relay is not the focus of this exercise.

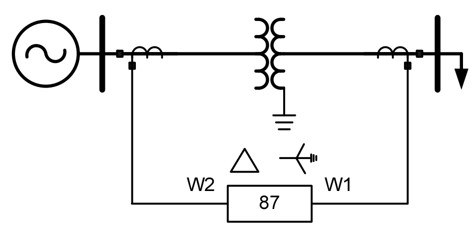
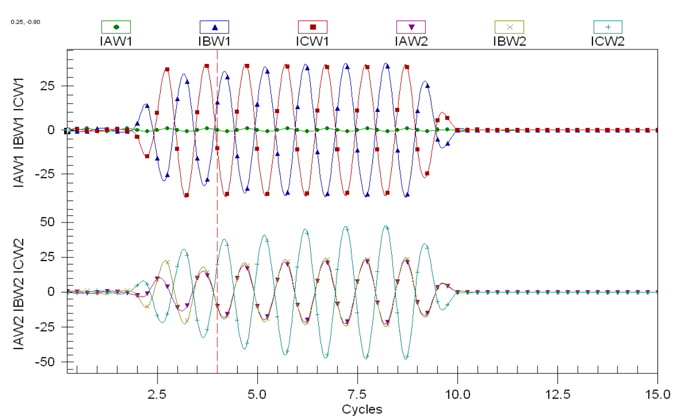


Fig. 12. Transformer current differential relay protecting a delta-wye transformer

IX-a What type of fault is this? Assuming a radial system, is the fault internal or external to the zone of protection?

It is a B-phase-to-C-phase fault on the wye side of the transformer. The fault is external because both relay CTs see fault current.



IX-b The transformer is connected to the system as shown in Fig. 13. Do we expect the currents on the delta side to lead or lag the currents on the wye side?

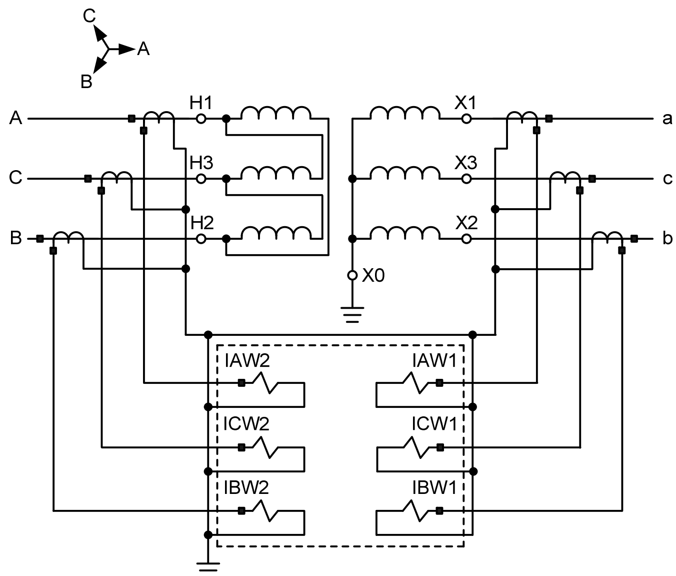
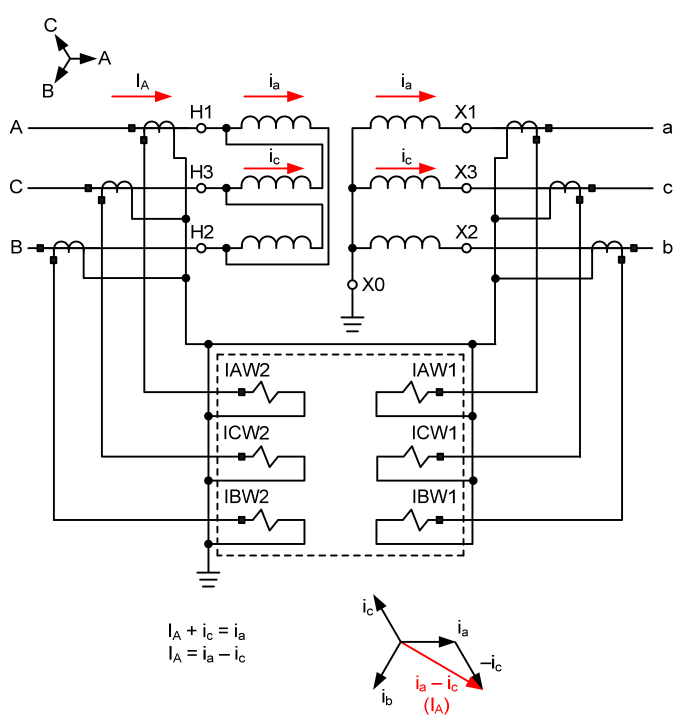


Fig. 13. Transformer phase-to-bushing connections

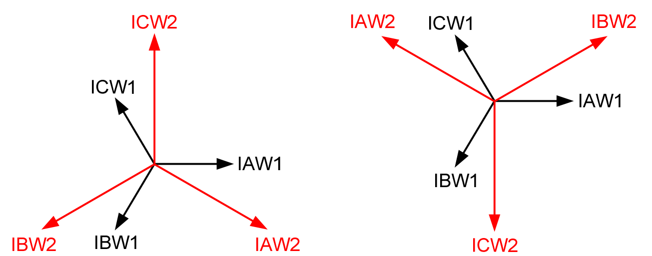
We can easily trace through any transformer connection to derive the lead/lag relationship between currents on either side. First, assume current flow through the transformer from the delta to the wye side. Knowing that the individual phase windings of the delta side are magnetically coupled to the individual windings of the wye side and assuming a transformer ratio of 1:1, we can conclude that the currents through them are the same. We can then write KCL equations to derive the currents on the phases coming into the delta winding. The derivation of this KCL equation for the delta-side C-phase current is shown in the following figure.



The result shows that the Winding 2 (delta) currents lag the Winding 1 (wye) currents by 30 degrees.

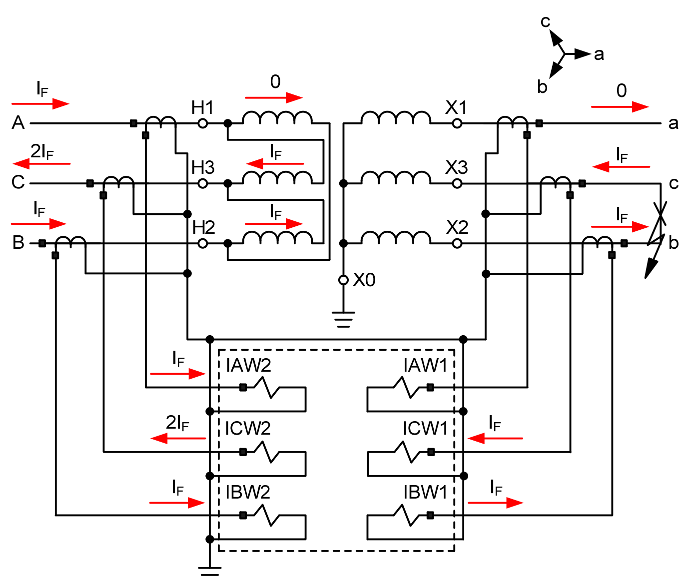
IX-c Draw the phasors for the prefault currents expected on the system as well as the phasors coming into the relay.

The prefault current phasors are shown in the following figure, with Winding 1 being the wye side and Winding 2 being the delta side. The left diagram shows the prefault current seen on the system, and the right diagram shows the prefault currents seen by the relay. Note that the Winding 2 currents are 180 degrees out of phase from the Winding 1 currents when seen by the relay because the CT polarity of Winding 2 is opposite that of Winding 1.

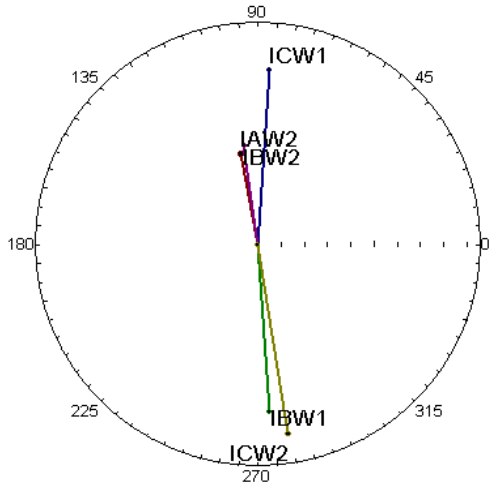


IX-d Draw the phasors expected on the system as well as coming into the relay during the fault. Does this match what the event shows?

Working from right to left, we can trace the fault through the transformer in a way similar to what we did with the prefault currents. This is shown in the following figure, assuming load is negligible on the unfaulted phases and the transformer ratio is 1:1.

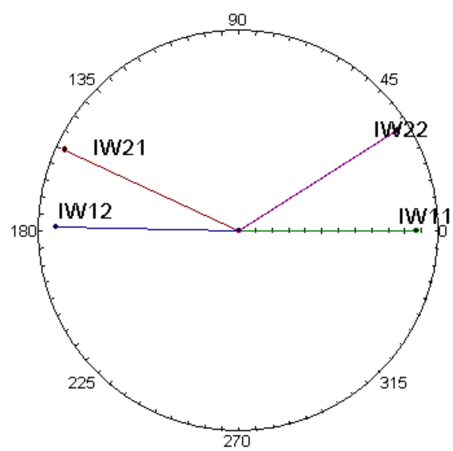


The phasors in the event, shown in the following figure, match what we expect to see. IBW1 is 180 degrees out of phase with ICW1, and IAW2 and IBW2 are in phase. ICW2 is 180 degrees out of phase with and twice the magnitude of IAW2 and IBW2.

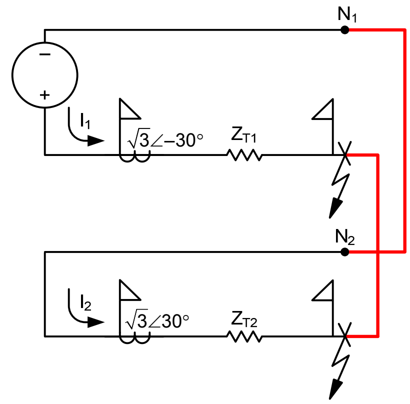


IX-e Look at the sequence phasors in the event. Derive these phasors by drawing the sequence network of the fault.

The sequence current phasors during the fault are shown in the following figure. Note that we must select an A-phase base when viewing the sequence phasors because this is a B-phase-to-C-phase fault.



The following figure shows the sequence network diagram for this fault, with the flags marking the locations of the CTs on either side of the transformer. Because this is a phase-to-phase fault, the positive- and negative-sequence networks are connected in parallel.



It is important to note that this network represents a phase-to-phase fault on the wye side of the transformer. This means we need to solve for the sequence currents going through the wye side first and then apply a delta transformation to solve for the currents going through the delta side.

Because this is a B-phase-to-C-phase fault, we need to use an A-phase base, so our transformation equation becomes:

$$\begin{bmatrix} I_{0Y} \\ I_{1Y} \\ I_{2Y} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_{AY} \\ I_{BY} \\ I_{CY} \end{bmatrix}$$

$$I_{AY} = 0,\ I_{BY} = -I_{CY}$$

$$I_{0Y} = 0$$

$$I_{1Y} = \frac{1}{3}\left(\alpha I_{BY} + \alpha^2 I_{CY}\right) = \frac{1}{3}\left(\alpha I_{BY} - \alpha^2 I_{BY}\right) =$$

$$\frac{1}{3}\left(\alpha - \alpha^2\right) I_{BY} = \frac{1}{3}\left(\sqrt{3} I_{BY} \angle 90\right)$$

$$I_{2Y} = \frac{1}{3}\left(\alpha^2 I_{BY} + \alpha I_{CY}\right) = \frac{1}{3}\left(\alpha^2 I_{BY} - \alpha I_{BY}\right) =$$

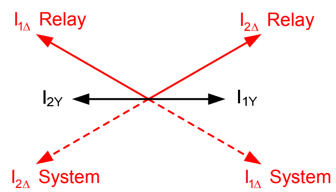
$$\frac{1}{3}\left(\alpha^2 - \alpha\right) I_{BY} = \frac{1}{3}\left(\sqrt{3} I_{BY} \angle -90\right)$$

$$I_{1Y} = -I_{2Y}$$

This solves for the sequence currents on the wye side of the transformer. In the first figure in this answer, we can see that the positive- and negative-sequence phasors on the wye side (Winding 1) are equal in magnitude and 180 degrees out of phase, which is expected from the equations we derived. We can also see that the positive-sequence phasor is about $\sqrt{3}/3$ times higher than the B-phase current on the wye side and leading by 90 degrees. Likewise, the negative-sequence phasor is about $\sqrt{3}/3$ times higher than the B-phase current on the wye side and lagging by 90 degrees.

Earlier, we derived that the currents on the delta side of the transformer are going to be $\sqrt{3}$ greater in magnitude and lagging the wye-side currents by 30 degrees in the positive sequence (leading by 30 degrees in the negative sequence). We represented this in the previous figure by taking the currents through the networks and transforming them through a CT that applies the appropriate magnitude increase and angle shift. The sequence currents seen on the delta side are after the CT transformation.

We can then draw the sequence phasors that we expect to see on the system, as shown in the following figure. Notice that $I_{1\Delta}$ is $\sqrt{3}$ greater in magnitude and lags I_{1Y} by 30 degrees, while $I_{2\Delta}$ is $\sqrt{3}$ greater in magnitude and leads I_{2Y} by 30 degrees. Due to CT polarity, we rotate the delta currents on the system by 180 degrees to see what the relay sees.



This matches the sequence phasors from the event, as shown in the first figure in this answer. Note that in the event, the delta currents are not $\sqrt{3}$ greater in magnitude than the wye currents, as we expect. This is because the $\sqrt{3}$ difference is based on an assumed transformer turns ratio of 1:1. See Example 7 for an explanation of how the transformer turns ratio affects the current magnitude relationship between the wye and delta sides.

IX-f Using the sequence components, work backwards to derive the phase fault currents on the delta and wye sides of the transformer.

Because we are using the A-phase as a base, our transformation equation is as follows:

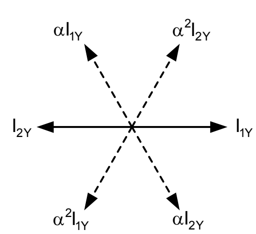
$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$I_A = I_0 + I_1 + I_2$$

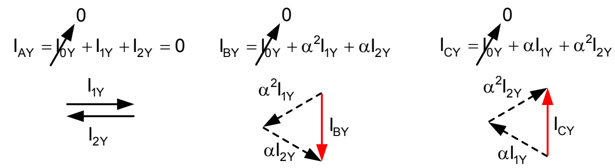
$$I_B = I_0 + \alpha^2 I_1 + \alpha I_2$$

$$I_C = I_0 + \alpha I_1 + \alpha^2 I_2$$

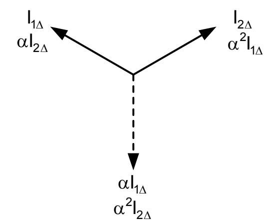
These computations can be done mathematically or graphically. The graphical method is shown here. For the wye currents, we first draw the sequence phasors shifted by α and α^2 that we will need, as shown in the following figure.



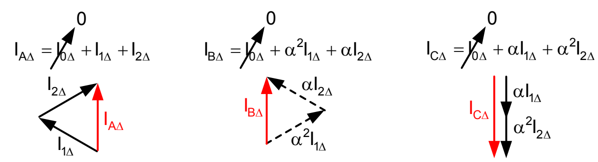
We then apply the transformations, as shown in the following figure.



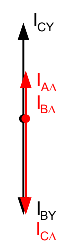
For the delta currents, we first draw the sequence phasors shifted by α and α^2 that we will need, as shown in the following figure.



We then apply the transformations, as shown in the following figure.



Combining the results of the phase currents on the delta and wye sides, we get the phasors shown in the following figure. This matches the phase currents during the event, as shown in the second figure in the answer to Question IX-d.



X. References

[1] J. L. Blackburn and T. J. Domin, *Protective Relaying Principles and Applications*. CRC Press, Boca Raton, FL, 2007.

[2] P. M. Anderson, *Analysis of Faulted Power Systems*. Iowa State University Press, Ames, IA, 1973.

[3] J. D. Glover, M. S. Sarma, and T. J. Overbye, *Power System Analysis and Design (4th Edition)*. Thomson Learning, Toronto, ON, 2008.

[4] E. O. Schweitzer, III and Stanley E. Zocholl, "Introduction to Symmetrical Components," proceedings of the 58th Annual Georgia Tech Protective Relaying Conference, Atlanta, GA, April 2004.

[5] Westinghouse Electric Corporation, Relay-Instrument Division, *Applied Protective Relaying*. Newark, NJ, 1976.

[6] C. F. Wagner and R. D. Evans, *Symmetrical Components*. McGraw-Hill, New York, 1933.

[7] D. Costello, "Event Analysis Tutorial, Part 1: Problem Statements," January 2012. Available: http://www.selinc.com.

[8] D. Costello, "Event Analysis Tutorial, Part 2: Answer Key," January 2012. Available: http://www.selinc.com.

[9] The Electricity Training Association, eds., *Power System Protection Volume 1: Principles and Components*. The Institution of Electrical Engineers, London, UK, 1995.

[10] P. K. Maezono, E. Altman, K. Brito, V. Alves dos Santos Mello Maria, and F. Magrin, "Very High-Resistance Fault on a 525 kV Transmission Line – Case Study," proceedings of the 35th Annual Western Protective Relay Conference, Spokane, WA, October 2008.

[11] D. A. Tziouvaras, J. Roberts, and G. Benmouyal, "New Multi-Ended Fault Location Design for Two- or Three-Terminal Lines," proceedings of the 7th International Conference on Developments in Power System Protection, Amsterdam, Netherlands, April 2001.

[12] A. Amberg, A. Rangel, and G. Smelich, "Validating Transmission Line Impedances Using Known Event Data," proceedings of the 65th Annual Conference for Protective Relay Engineers, College Station, TX, April 2012.

XI. Biographies

Ariana Amberg earned her BSEE, magna cum laude, from St. Mary's University in 2007. She graduated with a Masters of Engineering in Electrical Engineering from Texas A&M University in 2009, specializing in power systems. Ariana joined Schweitzer Engineering Laboratories, Inc. in 2009 as an associate field application engineer. She has been an IEEE member for 9 years.

Alex Rangel received a BSEE and an MSE from The University of Texas at Austin in 2009 and 2011, respectively. In January 2011, Alex joined Schweitzer Engineering Laboratories, Inc., where he works as an associate field application engineer. Alex is currently an IEEE member.

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