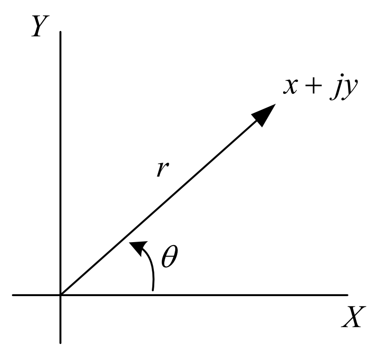
# PROT 301: Precourse Review

## Complex Numbers

A complex number is an ordered pair of real numbers typically represented as $x + jy$. The first element, $x$, is the real part; the second element, $y$, is the *imaginary* part; and $j$ is the square root of –1. The form, $x + jy$, is called the rectangular form (or the algebraic form).

Alternatively, a complex number can be represented in polar form. The polar form consists of a magnitude and an angle, such as $r\angle\theta$, where r is equal to the $\sqrt{x^2 + y^2}$ and $\theta$ is equal to the arctan($y/x$). Expressed another way, $x + jy = r(\cos\theta + j\sin\theta)$. The latter form is also called trigonometric form.




For example, if $x$ = 10 and $y$ = 8, then:

$$r = \sqrt{x^2 + y^2} = \sqrt{10^2 + 8^2} = \sqrt{100 + 64} = \sqrt{164} = 12.8$$
$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{8}{10}\right) = \arctan(0.8) = 38.7°$$
$$x = r \bullet \cos(\theta) = 12.8 \bullet \cos(38.7°) = 10$$
$$y = r \bullet \sin(\theta) = 12.8 \bullet \sin(38.7°) = 8$$

Another useful term is complex conjugate. The complex conjugate is easily obtained and can be expressed in rectangular or polar form. In rectangular form, the complex conjugate of $x + jy$ is $x - jy$. In polar form, the complex conjugate of $r\angle\theta = r\angle{-\theta}$.

Addition and subtraction of complex numbers is best achieved using the rectangular form, where

$$\begin{array}{r} (x+jy) \\ +\underline{(m+jn)} \\ (x+m)+j(y+n) \end{array}$$

and

$$\begin{array}{r} (x+jy) \\ -\underline{(m+jn)} \\ (x-m)+j(y-n) \end{array}$$

For example, if $\overline{A}=(7+j\,9)$ and $\overline{B}=(3+j\,5)$, then:

$$\overline{A}+\overline{B}=(7+3)+j\,(9+5)=10+j\,14 \quad \text{and}$$

$$\overline{A}-\overline{B}=(7-3)+j\,(9-5)=4+j\,4$$

Multiplying two complex numbers is best achieved using the polar form of the numbers.

Given:

$$\overline{A}=A\underline{/\alpha} \qquad \overline{B}=B\underline{/\beta}$$

Then:

$$\overline{A}\bullet\overline{B}=A\bullet B\underline{/(\alpha+\beta)}$$

$$\frac{\overline{A}}{\overline{B}}=\frac{A}{B}\underline{/(\alpha-\beta)}$$

$$\overline{A}\bullet\overline{A}^{*}=A^{2} \qquad \text{where } \overline{A}^{*} \text{ is the complex conjugate of } \overline{A}$$

$$\overline{A}^{n}=A^{n}\underline{/(n\bullet\alpha)}$$

$$\sqrt[n]{\overline{A}}=\sqrt[n]{A}\underline{/(\alpha+2k\pi)/n} \quad k=0,1,\ldots,n-1$$

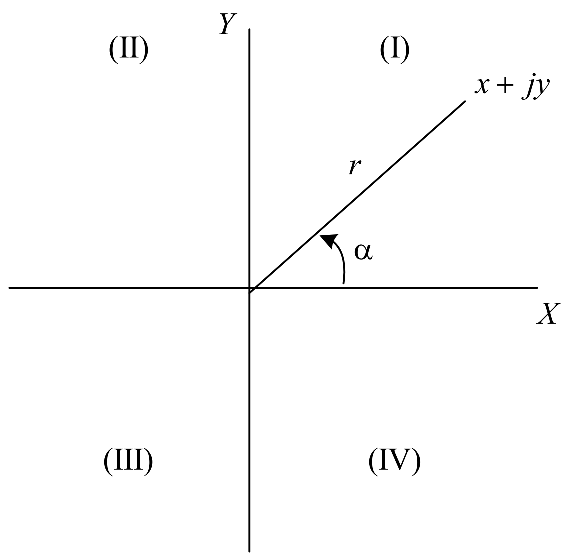
For example, if $\overline{A}=64\angle 25^{\circ}$ and $\overline{B}=5\angle 60^{\circ}$, then:

$$\overline{A}\bullet\overline{B}=64\bullet 5\angle(25+60)^{\circ}=320\angle 85^{\circ} \quad \text{and}$$

$$\frac{\overline{A}}{\overline{B}}=\frac{64}{5}\angle(25-60)^{\circ}=12.8\angle-35^{\circ}$$

## Trigonometry

Within the $X$-$Y$ plane, there are four quadrants: I, II, III, and IV. Quadrant I contains points with positive values of $X$ and $Y$. Quadrant II contains points with negative $X$ and positive $Y$. Quadrant III contains points with negative $X$ and $Y$. Quadrant IV contains points with positive $X$ and negative $Y$.




Useful definitions:

$$\sin\alpha = \frac{y}{r} \qquad \cos\alpha = \frac{x}{r} \qquad \tan\alpha = \frac{y}{x}$$

## Matrix Math

Given:

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \bullet \begin{bmatrix} D \\ E \\ F \end{bmatrix}$$

Then:

$$A = k_{11} \bullet D + k_{12} \bullet E + k_{13} \bullet F$$
$$B = k_{21} \bullet D + k_{22} \bullet E + k_{23} \bullet F$$
$$C = k_{31} \bullet D + k_{32} \bullet E + k_{33} \bullet F$$

For example:

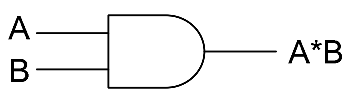
$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \\ 10 & 9 & 3 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Then:

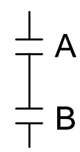
$$A = 3 \bullet 2 + 5 \bullet 3 + 7 \bullet 4 = 49$$
$$B = 4 \bullet 2 + 6 \bullet 3 + 8 \bullet 4 = 58$$
$$C = 10 \bullet 2 + 9 \bullet 3 + 3 \bullet 4 = 59$$

## Digital Logic

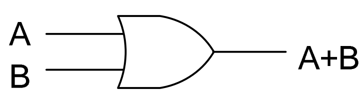
AND gate



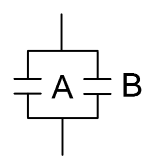

| A | B | A*B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



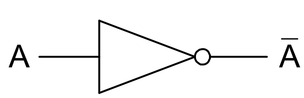

OR gate



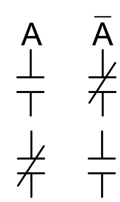

| A | B | A+B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |




NOT gate




| A | $\bar{A}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |



# Per Unit

The per-unit value of any quantity is defined as the ratio of the quantity to its base value and is expressed as a decimal. Per-unit methods provide easier calculations since the impact of voltage transformations does not have to be considered in the calculation. Additionally, relative values of quantities are easily discernable. Voltage, current, kilovolt-ampere, and impedance are interrelated and selection of base values for any two will determine the base values for the remaining two. Typically, the voltage and kilovolt-ampere base values are selected. As a general rule, the voltage base is the nominal voltage of the circuit. For system studies, the kilovolt-ampere base is often set at 100,000 kVA or 100 MVA. However, when not using a computer program, it may be beneficial to set the kilovolt-ampere base the same as the rating of a major piece of equipment, such as the power transformer that is central to the calculations.

$$\text{Base current, A} = \frac{\text{base kVA}_{1\varphi}}{\text{base voltage, kV}_{\text{LN}}} = \frac{\text{base kVA}_{3\varphi}}{\sqrt{3} \bullet \text{base voltage, kV}_{\text{LL}}}$$

$$\text{Base impedance, Z} = \frac{\text{base voltage, V}_{\text{LN}}}{\text{base current, A}} = \frac{\left(\text{base voltage, kV}_{\text{LN}}\right)^2}{\text{base MVA}_{1\varphi}} = \frac{\left(\text{base voltage, kV}_{\text{LL}}\right)^2}{\text{base MVA}_{3\varphi}}$$

$$\text{Per unit Z}_{\text{new}} = \text{per unit Z}_{\text{old}} \bullet \left(\frac{\text{base kV}_{\text{old}}}{\text{base kV}_{\text{new}}}\right)^2 \bullet \frac{\text{base kVA}_{\text{new}}}{\text{base kVA}_{\text{old}}}$$

## *Example 1:*

$$\text{If } V_{\text{base}} = 115 \text{ kV and } S_{\text{base}} = 100 \text{ MVA, then}$$

$$I_{\text{base}} = \frac{\text{base kVA}_{3\phi}}{\sqrt{3} \bullet \text{base voltage, kV}_{\text{LL}}} = \frac{100{,}000}{\sqrt{3} \bullet 115} = 502 \text{ A}$$

$$Z_{\text{base}} = \frac{\left(\text{base voltage, kV}_{\text{LL}}\right)^2}{\text{base MVA}_{3\phi}} = \frac{115^2}{100} = 132.25\ \Omega$$

## *Example 2:*

$$\text{If } V_{\text{base}} = 13.8 \text{ kV and } S_{\text{base}} = 20 \text{ MVA, then}$$

$$I_{\text{base}} = \frac{\text{base kVA}_{3\phi}}{\sqrt{3} \bullet \text{base voltage, kV}_{\text{LL}}} = \frac{20{,}000}{\sqrt{3} \bullet 13.8} = 836.7 \text{ A}$$

$$Z_{\text{base}} = \frac{\left(\text{base voltage, kV}_{\text{LL}}\right)^2}{\text{base MVA}_{3\phi}} = \frac{13.8^2}{20} = 9.5\ \Omega$$